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NOTE.—The closing dates herein published are final except when names of prospective discussers are registered for special extension of time.

* Publication of closing discussion pending.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

DRAINAGE OF LEVEED AREAS IN MOUNTAINOUS VALLEYS

By Gordon R. Williams,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Various methods of disposing of the drainage from streams tributary to leveed areas are outlined in this paper. Brief consideration is given to the characteristics of these methods in fulfilling the design criterion that local drainage must be disposed of without causing damage appreciably greater than if the streams could flow unobstructed to the main river at low stage.

The details of a method of analyzing local hydrology and developing capacities of drainage structures under various conditions are presented. Graphs show volumes and rates of rainfall and runoff used in the design storms and floods, and relations between selected capacities and available storage for numerous designs for drainage structures.

Introduction

The drainage of leveed areas on the flood plains of deep valleys results in more problems, and fortunately more possible solutions to those problems, than does the drainage of other types of leveed areas. For example, in providing for local drainage of areas behind levees in the lower Mississippi River Valley, in general there is no alternative but to install pumping stations, as there is no opportunity for providing head or storage for other types of works. Also, the drainage of an urban area in relatively low country already provided with a storm-water drainage system can be disposed of only by a pumping station directly connected to the outfalls.

The problems of levee protection and subsequent local drainage in a mountainous valley may differ from those in level or rolling country in the following respects:

(1) Levee protection is usually justified only for urban areas, in which case right-of-way costs will be high;

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May, 1942. Opening discussion on this paper appears elsewhere in this issue.

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(2) The narrowness of the flood plain and the concentration of buildings limit the space for, and type of, structures to be built;

(3) The topography increases the drainage area tributary to the leveed area; and

(4) The topography requires that drainage be provided for streams whose hydrologic characteristics may differ from those of the protected areas.

Conditions (1) to (4) have been encountered in connection with flood-protection projects on the North and West branches of the Susquehanna River in Pennsylvania at the communities of Wilkes-Barre, Hanover Township, Kingston, Edwardsville, Plymouth, and Williamsport. The hydrologic procedures to be presented in this paper were developed particularly for the design of the protective works at Hanover Township, Plymouth, and Williamsport.

LOCAL DRAINAGE REQUIREMENTS

Authorized projects of the U. S. Engineer Department for the protection of communities specify the river against which protection is to be provided. The degree of protection afforded against the specified river is based on studies of past floods and economic considerations. The adopted project usually provides for a substantial freeboard against the confined flow of the greatest known flood. A corresponding degree of protection is not required against flows from tributaries that pass through leveed areas, but it is required that the protective works on the main river shall not aggravate conditions caused by floods on the tributary streams. If a community desires flood protection from a tributary stream, or any other local improvements not a part of an authorized project, it must obtain authorization from Congress, which in turn may request the U. S. Engineer Department to make additional investigations for such improvements. Flood protection from small tributaries is very rarely justified.

The problem of preventing conditions within the leveed area from being no worse than if the tributary stream could flow unobstructed to the river is a difficult one and requires the consideration of a variety of situations. It is obvious from a hydraulic standpoint that a perfect solution is not possible, but from a damage standpoint a satisfactory solution can be obtained. In other words, it is possible to dispose of interior runoff at a rate that will prevent local damage from being appreciably greater than with no levee and the river at low stage. Benefits resulting from the fact that the main river has been prevented from entering the leveed area do not affect the economics of the local drainage problem because the elimination of main river damage has already been considered in determining the justification for the entire project.

There are two conditions for which the effect of floods on local tributaries must be considered:

(*a*) When the main river is at low stage and normal drainage to the river can take place; and

(*b*) When the river is at or above flood stage and special drainage works must be provided.

On the North Branch of the Susquehanna River conditions (*a*) and (*b*) are largely confined to well-defined seasons and therefore require a detailed study of probable hydrologic conditions that are characteristics of those seasons.

METHODS OF DISPOSING OF LOCAL DRAINAGE

There are six basic methods of disposing of local drainage from tributaries passing through leveed areas. These methods are: (1) Levees or walls along the tributary stream, (2) culvert with floodgate, (3) pressure conduit, (4) diversion channel, (5) storage reservoir, and (6) pumping station. Any of these methods may be used alone, but two or more are customarily used in combination. The selection of any method or combination of methods is based on studies of comparative costs and consideration of special local conditions.

(*1*) *Levees or Walls Along the Tributary Stream.*—From the standpoint of simplicity this method is to be preferred. However, it may be relatively costly for small tributaries or those with a drainage area of less than about 50 sq miles. The levees or walls must be extended upstream along the banks of the tributary to high ground and must be at an elevation sufficient to provide freeboard under all reasonable combinations of coincident discharges at the outlet of the tributary. If the tributary flows through an urban area, right-of-way costs for levees or walls may be prohibitive. Some tributaries along the Susquehanna River enter the flood plain at right angles to the course of the main river and then flow parallel to the river, sometimes for several miles, before finally emptying into the river. Under such conditions levees along the banks of a tributary become a major project in themselves. Levees or walls may be constructed part way up a tributary to connect with other works, such as a pressure conduit or pumping station. Levees have been provided in the Susquehanna River projects only on Lycoming Creek at Williamsport, which has a drainage area of 269 sq miles, but they were considered for other tributaries much smaller in size.

(*2*) *Culvert with Floodgates.*—Relief culverts with floodgates are used for floods occurring when the river is at low stage. Floodgates will pass local drainage at any stage in the river, provided there is sufficient differential head resulting from ponding at the intake. The selected size of culvert depends upon the design inflow and the allowable ponding elevation at the entrance. The latter elevation is dependent on the damage that will result from ponding. Obviously it is impossible to eliminate all ponding at the levee and still obtain an effective head on the culverts. Where conditions require excessively large batteries of culverts, there may be justification for providing smaller openings in combination with a fuse plug in the levee that can be blown out if conditions warrant. Relief culverts and floodgates have been constructed or planned at every point of natural outlet at levees in the Susquehanna River projects.

(*3*) *Pressure Conduit.*—A pressure conduit may be used if there is sufficient head. The total energy head or difference in elevation between stage in the river and the energy elevation at the entrance must be sufficient to overcome losses in the conduit and to produce velocity of flow under various combinations of river stage and conduit discharge. To obtain the head it is usually necessary to place the entrance some distance from the main river, resulting in a reduction in the area diverted and an increase in the length of the conduit. Pressure conduits are most practicable where the flood plain is narrow so that the construction and right-of-way costs are at a minimum. Sometimes the conduit can be laid in the original stream channel, but if that channel has been used as an outlet for storm-water drains, a separate intercepting sewer must be built or connections with gates or valves provided in the conduit. Pressure conduits also require supplementary works consisting of a pumping station and a relief culvert. The pumping station is required to dispose of runoff from the undiverted area and runoff from the diverted area in excess of the capacity of the conduit. The latter runoff will usually occur during low stages in the river and will pass through the relief culverts. The only pressure conduit under construction in the Susquehanna River flood-control projects is on Toby Creek at Kingston, but others have been considered.

(*4*) *Diversion Channel.*—Diversion of tributary streams to a point outside of leveed areas is usually practicable only when a stream is close to the limits of the protected area. In such cases changing the natural course of a tributary is often the most economical procedure. However, if the fall from the point of diversion to the river is great, and if there are highways and railroad tracks to be crossed, the costs of drop structures and bridges may indicate that some other method is more economical. Diversion channels have been planned for Coal Creek at Plymouth (diverted area, 1.6 sq miles) and Millers Run at Williamsport (diverted area, 7.8 sq miles).

(*5*) *Storage Reservoirs.*—Under certain combinations of favorable conditions it may be economical to construct storage reservoirs to impound the runoff of tributaries during floods in the main river. Reservoirs may be located at levees or at higher elevations farther upstream. In order to satisfy the criterion adopted for the design of local drainage structures, the reservoir need not provide flood control during low stages in the main river and hence has only to provide storage for the limited period in which the relief culverts cannot operate. Such reservoirs may be effective with only 2 or 3 in. of storage instead of the 6 in. usually considered necessary for flood control. Reservoirs with dams must be constructed back in the hills where dam sites are available, in which case there usually will be a considerable uncontrolled area, the runoff from which must be pumped. In addition, the usual relief culverts must be provided for floods not controlled by the reservoir. Reservoirs with dams have been considered but never found to have been justified in the solution of levee drainage problems in the Susquehanna River Basin; but reservoirs adjacent to levees have played an important part in the solution of such problems.

(*6*) *Pumping Station.*—If the hydrographs for design floods are routed through existing ponding areas and constrictions in the channel and valley of a tributary stream, it often will be found that, with only a relatively small rate of outflow, water-surface elevations can be kept close to conditions prior to the construction of the levee. Hence, a pumping station is used more than any other method to satisfy the drainage criterion. Such pumping installations usually take full advantage of the modifying effect of available storage adjacent to the levee. Such storage is usually in low marginal areas where development has been limited and where damage will be slight. The chance that such areas may be developed and filled after the levee is built must be considered and requires an appraisal of possible future development. Relief culverts must be provided in connection with pumping stations to dispose of runoff that may occur when the river is at low stage. A total of eleven pumping stations is planned, for unsewered or partly sewered areas on Susquehanna River levee projects in Pennsylvania. An additional eleven pumping stations are planned for completely sewered areas.

Local Hydrology

The determination of appropriate rates and volumes of runoff to be used in the determination of the capacities of drainage structures is exceedingly difficult, especially when there are no available records in the immediate vicinity of a project. It requires a consideration of the seasons in which different conditions may occur, an analysis of rainfall and runoff characteristics of these seasons, and a determination of the runoff characteristics of each sub-area to be drained. In usual storm-water drain design it is considered necessary only to determine peak rates of runoff, and the effect of storage is neglected. In the design of drainage structures for natural areas behind levees, it is necessary to route complete hydrographs through existing and proposed ponding areas in order to determine the modified peak rates for which to provide. The hydrologic problem is to determine, as well as to route, hydrographs that have a reasonable probability of occurrence under different conditions.

Basic Data.—It is desirable to have actual records or estimates of stage in the main river, of runoff from the tributaries, and of local rainfall and snowfall. If the main stream is important as is the Susquehanna River, long records of stage are usually available or can be determined from near-by places by means of gage-relation curves. Continuous records of stage on the North Branch at Wilkes-Barre began in 1896 and on the West Branch at Williamsport in 1895.

The tributary creeks, on the other hand, are small and have been considered too unimportant to be gaged. On some there are a few flood marks that make it possible to estimate the maximum floods known to the residents. For the most part there are no records on which to base times of concentration or unit-hydrograph characteristics.

Rainfall records at nonrecording gages are available at many places in the Susquehanna River Basin, and some of the records have been kept for 45 years or more. Unfortunately, there are only two recording gages in the basin in Pennsylvania upon which to determine the frequency of rainfall intensities of short durations. These gages are located at Harrisburg and Scranton. All these records have been analyzed and form the initial basis for determining the capacity of drainage structures.

In order to obtain more reliable data on the runoff characteristics of the small drainage areas adjacent to active or proposed flood-control projects, the

U. S. Engineer Department has established, in cooperation with the U. S. Geological Survey, a number of gaging stations on small areas. The smallest areas on which gaging stations were established are listed in Table 1.

TABLE 1.—GAGING STATIONS ON SMALL AREAS IN THE SUSQUEHANNA RIVER BASIN IN PENNSYLVANIA

No.	Stream	Place	Area[a]
1	Paxton Brook	Harrisburg	11.2
2	Solomon Creek	Wilkes-Barre	15.4
3	Hageman Run	South Williamsport	6.3
4	Grafius Run	Williamsport	3.1

[a] Drainage area in square miles.

A recording rain gage was also established on the headwaters of Solomon Creek. The new system of recording rain gages established by the Federal State Flood Forecasting Service of Pennsylvania is proving of increasing value in connection with newly established stream gaging stations. These streamflow and rainfall gages, even when in operation for only one flood season, often give information that results in the saving of large sums of money in the design of drainage structures.

Flood Season.—The flood season on the Susquehanna River was determined from an analysis of stage records at Wilkes-Barre. The number of flood peaks in the record above damage stage are as follows:

Month	No.	Month	No.
October	3	April	6
November	3	May	1
December	1	June	0
January	3	July	2
February	4	August	0
March	13	September	0

It is evident that 83% of the peak damage stages have occurred in the season from November to April, inclusive, and that March and April are the most severe flood months. The purpose of defining the flood season is to determine when the drainage structures will be used and what rainfall and runoff conditions can be expected during the period of use. Runoff from snow cover can be expected throughout the selected season. If the season had been extended farther to include October and May, the absence of snow runoff and the increased losses from infiltration would counteract the higher rainfall rates to be expected during those months.

Rainfall in Flood Season.—It should be emphasized that the drainage areas under consideration are so small that point rainfall data are sufficient for winter storms and lead to a desirable factor of safety for summer storms, and that area-depth relations need not be introduced to add to the complexity of the problem. A study was made first of 1-day and 2-day peak rainfalls in the flood season for eighteen stations in northeastern Pennsylvania. The excessive rainfalls were tabulated by months and then combined into a seasonal record. The frequency of the various rainfall depths was computed and plotted and mean curves drawn. A summary of the results is shown in Table 2.

Considering that the rainfall depths were obtained from calendar-day records of rainfall, and that the actual durations of the rainfalls probably varied over a wide range, there is a remarkable correlation between the results. This is due in part to the fact that winter storms cover wide areas and cause a relatively uniform distribution of precipitation. Such a correlation, which has been anticipated by other investigators,[2] should not be misconstrued as indicating a high degree of accuracy in the frequency-depth relations. Some other period of record might show a similar correlation but not the same frequency-depth relations.

TABLE 2.—SUMMARY OF RAINFALL RECORDS FOR THE SEASON, NOVEMBER TO APRIL, INCLUSIVE

Station (Pennsylvania)	Elevation, in ft above mean sea level	Length of record, in years	Rainfall, in Inches[a]					
			5 Years		10 Years		15 Years	
			1-day	2-day	1-day	2-day	1-day	2-day
(*a*) SUSQUEHANNA RIVER BASIN								
Wilkes-Barre	544	44	1.8	2.3	2.0	2.7	2.1	2.9
Scranton	746	39	2.0	2.3	2.2	2.6	2.4	2.8
Forest City	....	18	1.9	2.4	2.2	2.7	2.3	2.9
Towanda	754	44	1.9	2.3	2.2	2.6	2.4	2.8
Montrose	1,656	36	1.7	2.0	2.0	2.3	2.1	2.5
Morris Run	....	14	2.2	2.9	2.7	3.2	2.9	3.5
Lawrenceville	1,000	43	1.9	2.3	2.2	2.7	2.4	2.9
Wellsboro	1,319	43	2.1	2.6	2.4	3.0	2.6	3.2
Ansonia	1,136	26	1.6	2.3	1.8	2.7	1.9	3.0
Williamsport	542	43	2.2	2.7	2.4	3.1	2.6	3.3
Lock Haven	554	43	2.0	2.4	2.2	2.8	2.3	3.0
Muncy Valley	864	28	2.2	2.7	2.5	3.1	2.7	3.4
Catawissa	520	28	2.0	2.6	2.3	2.9	2.5	3.2
(*b*) DELAWARE RIVER BASIN								
Pleasant Mount[b]	....	15	2.2	2.5	2.4	2.8	2.6	2.9
Freeland[b]	1,900	25	2.2	2.9	2.5	3.3	2.7	3.5
Gouldsboro	1,890	25	2.2	2.9	2.5	3.1	2.6	3.3
Mount Pocono	1,740	24	2.6	3.3	2.9	3.9	3.1	4.2
Stroudsburg	500	23	2.2	2.8	2.5	3.1	2.6	3.4

[a] Rainfall to be equaled or exceeded once in the period indicated. [b] On divide between Susquehanna River and Delaware River basins.

Because the critical rainfalls on small areas are those of high intensity and short duration, records of calendar-day rainfall have little application except where extensive flood storage is planned. It is believed, however, that these rainfalls can be used as an index of the possible regional variations in rainfalls of shorter durations. Furthermore, the data in Table 2 indicate that, for the winter season, the increase in depth of rainfall with decrease in frequency is relatively small. For example, the average increase in the expected depth of rainfall between the 5-yr and 10-yr frequencies or between the 10-yr and 15-yr frequencies is only about 10%. This characteristic of the winter storms is significant in the design of drainage structures and indicates that the degree of protection against interior flooding can be increased often without a large increase in expenditure.

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[2] "The Reliability of Rainfall Intensity-Frequency Determinations," by C. W. Thornthwaite, *Transactions*, Am. Geophysical Union, Pt. II, 1937, pp. 476–484.

In order to obtain data on the winter rainfalls of short duration, the original charts for the recording gages at Scranton and Harrisburg were studied. All peak rainfall depths for durations of from 5 min to 12 hr were determined. The rainfall depths for each duration were arranged in order of magnitude, and the frequency with which each value was equaled or exceeded was computed and mean curves drawn. Frequency curves for the Scranton record, which differ but little from those obtained from the Harrisburg record, are shown in Fig. 1. Total depths of rainfall, instead of hourly rates, were used because they can be applied directly to unit hydrographs and because depth-frequency curves when plotted on semilogarithmic paper indicate straight-line relations.

The computed frequencies for the rainfalls of all durations, including the 1-day and 2-day rainfalls, were determined by the formula

$$f = \frac{2N}{2m - 1} \qquad \text{(1)}$$

in which N = length of record in years; m = serial number of each item, when items are arranged in descending order of magnitude; and f = frequency, in years, with which any item will be equaled or exceeded. Eq. 1, which is based on the theory of least squares, has been universally accepted in estimating flood-peak frequencies but has been rarely used in determining rainfall frequencies. No doubt the reason is that rainfall records furnish more items for analysis, and investigators have considered that theoretical adjustments to available records were not necessary. The formula is just as applicable to one type of frequency analysis as it is to another, including the construction of duration curves.[3]

Design Storm for Flood Season.—In selecting a design storm there are two possible courses of procedure. One is to use the actual rainfalls from some severe storm of record in the flood season, and the other is to derive a synthetic storm from rainfalls of equal frequency. The first method may result in either under-design or over-design of drainage structures unless the characteristics of the design storm are compared carefully with many other storms, particularly those that have occurred during high stages in the main river. The second method, which was chosen for use in the Susquehanna Basin, results in a storm that is rarer than the indicated frequencies of the component rainfalls but eliminates to a large extent the uncertainties resulting from the use of a particular storm. This procedure has been suggested by other engineers.[4,5]

An examination of the rainfall depths in Table 2 indicated that the studies of the recording gage record at Scranton could be applied to areas in the vicinity of Wilkes-Barre. In applying the same studies to areas in the vicinity of Williamsport, the rainfalls of short duration were increased by the ratio between the Williamsport and the Scranton daily rainfalls of the same frequency.

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[3] "Duration Curves," by H. Alden Foster, *Transactions*, Am. Soc. C. E., Vol. 99 (1934), pp. 1213–1267.

[4] Discussion by Merrill M. Bernard of "Relation Between Rainfall and Run-Off from Small Urban Area," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 189.

[5] "Storage Basins as a Supplement to Storm Sewer Capacity," by John A. Rousculp, *Civil Engineering*, November, 1940, pp. 715–718.

The design storms were derived by combining rainfalls of equal frequency

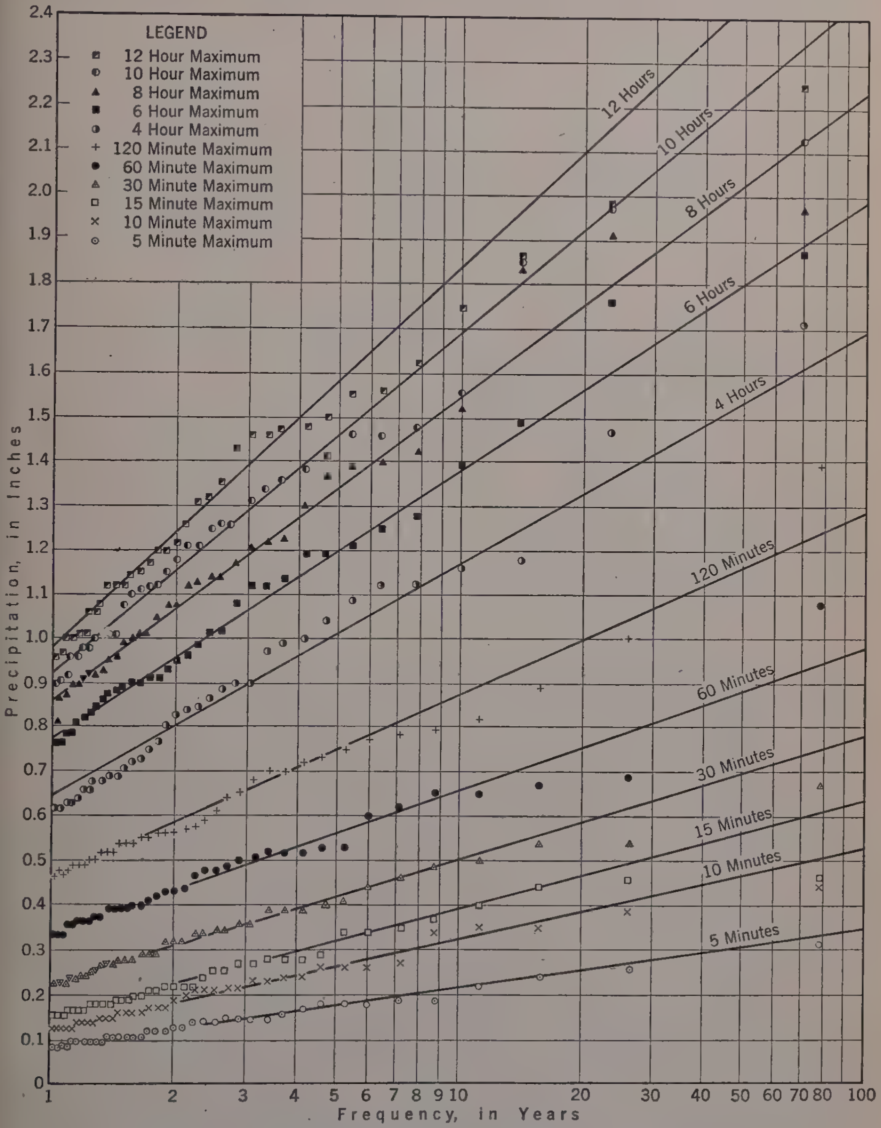

FIG. 1.—FREQUENCY ANALYSIS OF PRECIPITATION AT SCRANTON, PA., FOR THE SEASON NOVEMBER TO APRIL, INCLUSIVE

into a histogram which was nearly symmetrical and which had the maximum intensity at the midpoint. As 10-min unit hydrographs were used for areas less than about 8 sq miles, rainfalls in the design storms were computed for

10-min intervals. In northern Pennsylvania it was assumed that there would be 100% runoff in the winter season and that, in addition, there would be runoff from melting snow at the rate of 1 in. in 24 hr. The histograms for the design storms of different frequencies are shown in Fig. 2. Mass curves of rainfall plus snow melt are shown in Fig. 3.

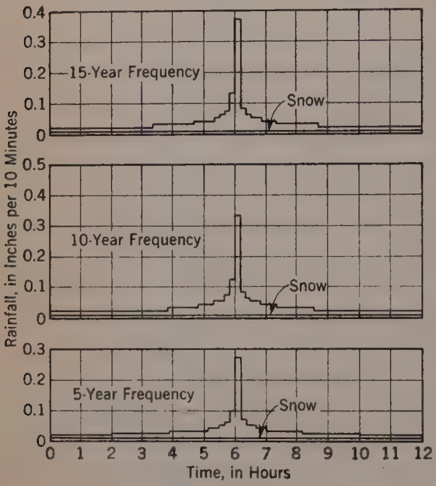

FIG. 2.—DESIGN STORMS FOR THE SEASON NOVEMBER TO APRIL, INCLUSIVE

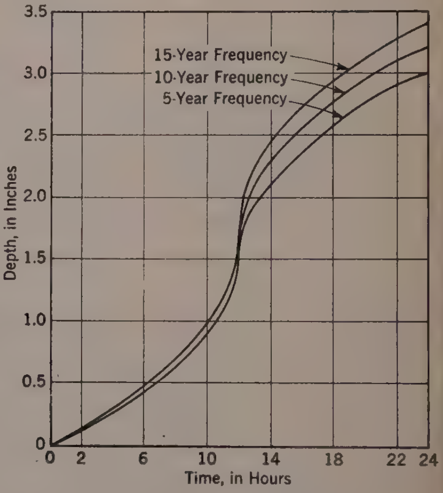

FIG. 3.—MASS CURVES OF RAINFALL PLUS SNOW MELT FOR DESIGN STORMS

At this point it is well to raise the question as to whether the design storm should be based on all rainfalls in the flood season or on rainfalls that have occurred coincidental with flood stage in the main river. Theoretically the latter rainfalls should be used, but practically they do not form a reliable basis for design in the locality under consideration. Using available records of daily rainfall it was found, for example, that the 10-yr, 1-day rainfall for the entire flood season was equaled or exceeded about once in 35 or 40 years in coincidence with high stage in the river. A similar study was conducted taking the hourly rainfalls at Scranton that occurred during high stage at Wilkes-Barre. The results were not conclusive and no reliable frequency curve could be drawn. It was noted, however, that many intense hourly rainfalls did coincide with high stage in the river. In order to determine frequencies of rainfalls coincident with high river stages, which would be as reliable as the frequencies of rainfall throughout the entire flood season, it would be necessary to have a record of coincident events three or four times as long as the available rainfall record. It should also be emphasized that winter floods on the river and on the small tributaries do not have independent meteorological causes. The winter storm is widespread and is usually accompanied by warm temperatures that cause melting of snow throughout the storm area. Peaks of floods on the tributary streams will rarely coincide with peaks on the main stream, but tributary peaks will often coincide with high stages on the main stream. Furthermore, the phenomenon of two or more storms in succession is a common occurrence in the Susquehanna Basin, as in March, 1936. In the flood of March-April, 1940, there were three distinct flood rises above the damage stage at Wilkes-Barre in a period of 12 days.

On rivers with drainage areas in excess of about 10,000 sq miles, or smaller rivers that take many days to reach their flood crests, it may be safe to base the design of drainage structures on known rainfalls coincident with high stages. Such studies have been conducted by the U. S. Engineer Department for cities along the Ohio River, where there are long records of river stage and coincident rainfall. On the Ohio River below Pittsburgh, Pa., it can be assumed safely that there is no relation between local runoff and floods on the main river. The duration of high stages is much longer than on the Susquehanna River at Wilkes-Barre, and therefore there is more opportunity to record coincident rainfall.

Design Hydrograph for Flood Season.—Flood flows that would result from the design storms were determined by applying rainfall plus assumed snow melt to the unit hydrographs for the various local areas. As mentioned before, the entire storm rainfalls were considered to represent rainfall excess, and infiltration was assumed to be zero. The unit hydrographs were obtained from actual records of runoff where available, but most of the unit hydrographs were derived synthetically.

Unit Hydrograph.—The basic procedure in determining a synthetic unit hydrograph was first to derive a unit hydrograph for a rainfall period equal to the time of concentration of an area and then to modify that hydrograph so that it was applicable to the rainfall period used in the design storm, which was 10 min for most areas. The basic theory involved has received consideration elsewhere.[6]

Determination of the time of concentration was a problem in itself. A procedure developed by Z. P. Kirpich,[7] Jun. Am. Soc. C. E., was used. Briefly, this method is to compute, for areas with known times of concentration, correlation factors based on length of stream channel, slope of channel, and average slope of basin. These factors are plotted against known times of concentration, and the resulting curve is used to determine times for other areas where only the correlation factor is known. The results from the data plotted by Mr. Kirpich were extrapolated to give a basis for determinations applicable to the larger and more mountainous areas in the Susquehanna Basin. The extrapolation was checked closely from field data obtained from one mountainous area of about 15 sq miles. It is realized that no results of great precision are obtained by such a procedure. However, it should be more reliable than mere guessing at velocities of overland and stream-channel flow. Furthermore, when more field data become available, it is believed that extension of the same procedure will yield results of greater precision.

The peak discharge of the unit hydrograph for a rainfall period equal to the time of concentration was determined by use of the principle that, if the rainfall excess continues for a period equal to the time of concentration, the rate of runoff will equal the rate of rainfall excess. The familiar rational formula

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[6] "Analysis of Run-Off Characteristics," by Otto H. Meyer, *Transactions*, Am. Soc. C. E., Vol. 105 (1940), pp. 83–141.

[7] *Civil Engineering*, June, 1940, p. 362.

$$Q = C\,i\,A \quad (2)$$

is based on this theory. If C, the runoff coefficient, is considered equal to unity, the formula can be considered to represent rainfall excess. As the unit hydrograph represents 1 in. of runoff from a uniform rainfall, the rate of rainfall excess equals

$$i = \frac{1}{T_c} \quad (3)$$

in which T_c equals the time of concentration in hours. As T_c for small areas is usually expressed in minutes, i equals $\frac{60.}{T_c}$ The area, A, is expressed in acres, and the peak rate of discharge, Q_p, for the unit hydrograph is expressed by the formula

$$Q_p = \frac{60\,A}{T_c} \quad (4)$$

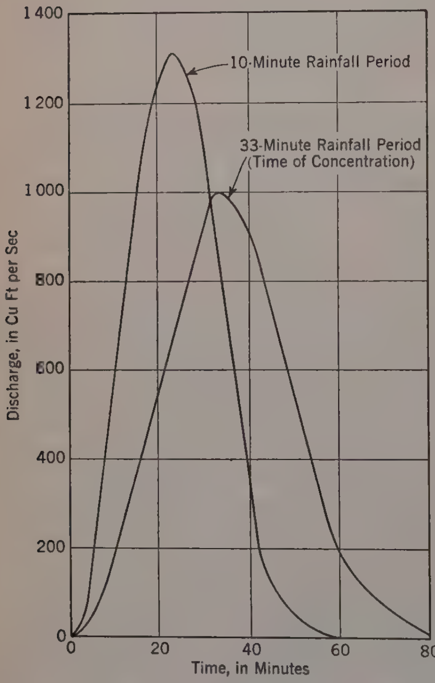

FIG. 4.—TYPICAL UNIT HYDROGRAPHS; STREAM: FOX HOLLOW RUN, WILLIAMSPORT, PA.; DRAINAGE AREA: 548 ACRES

The writer believes that this theoretical rate of discharge is never reached in the time T_c for natural areas having appreciable channel and depression storage. However, such an assumption is conservative and must be used until future studies indicate how the storage effect can be evaluated.

To estimate the shape of rising and falling limbs of the synthetic unit hydrograph, contours representing equal times of travel to the outlet were drawn. From these contours a time-area graph was constructed, and the ordinates to the unit hydrograph were made proportional to this graph. Of course, the use of time contours and time-area graphs is not new, but the relation between those principles and that of the unit graph appears to have been first demonstrated by Merrill Bernard,[8] M. Am. Soc. C. E., and later developed by others,[6] who introduced the unit hydrograph for the time of concentration. In order to conform more nearly to natural unit hydrographs, the falling limbs were arbitrarily flattened so that the time from the peak to zero flow was equal to $\frac{3}{2}$ T_c. Typical unit hydrographs for an area with a time concentration of 33 min are shown in Fig. 4.

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[8] "An Approach to Determinate Stream Flow," by Merrill Bernard, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 349.

Maximum Probable Storm and Flood.—In order to provide adequate relief openings when the main river is at low stage, a maximum probable storm and flood were computed for each area. The maximum probable storm would be of the summer cloudburst type. The rainfalls for such a storm were determined from an envelope of maximum known rainfalls for northeastern United States, which has been published previously.[9] Using this curve a synthetic storm was constructed in the same manner as the winter design storm. The maximum probable flood hydrograph was derived by deducting an estimated infiltration loss from the rainfalls and then applying the net depths to the unit hydrographs. Studies of storms on larger areas where stream-flow records were available indicated that an infiltration rate of 0.3 in. per hr was conservative for summer storms. This rate assumes that there has been antecedent rainfall, and much higher rates would prevail if the rain fell on dry ground.

No claim is made that the hydrologic analyses presented herein represent perfection in such studies, but rather that they represent a workable and reasonable solution to a difficult engineering problem. Further refinements probably could not be justified from the standpoint of the reliability of the available basic data and from the probable savings in the design of the structures.

DETERMINATION OF CAPACITY OF DRAINAGE WORKS

The hydrographs for the winter design flood and the maximum probable flood, determined as described in the previous section, were routed through all areas of appreciable existing and proposed storage. Most of the effective storage is found to be in the flood-plain areas. In the case of a proposed diversion channel or pressure conduit there is usually little or no storage above the entrance to modify the inflow hydrograph for the diverted area and to reduce the required capacity. On the other hand, structures such as pumping stations and relief culverts, which are located at levees, take full advantage of all modifying storage in the flood plain, which reduces the required capacity without violating the design criterion. Of course, the area drained is greatest at the levee, but the increase in the volumes of runoff is usually offset by the increase in ponding facilities.

It was considered reasonable for the drainage works to have a capacity sufficient to dispose of runoff from the 10-yr winter flood without causing damage in the flood plain appreciably greater than if water could flow freely to the river at low stage.

In order to determine the incremental increase in damage that would result from various ponding elevations at the levee, stage-damage curves were plotted for each area affected. Data for these curves were obtained from detailed surveys for the entire flood plain. The latter surveys had been made previously in order to determine the justification for each local protection project as a whole.

Because of the characteristics of the winter storms, already discussed in this paper, the selection of the rainfall frequency as between 5, 10, or 15 years usually does not have a very material effect on the size of drainage structures. The greater the flood-plain storage, the less the variations in inflow rates affect the capacity of structures. Fig 5 shows the relations between pumping capacity and storage for a number of designs based on 10-yr winter rainfalls. Local conditions affect the flood routing and in turn affect the selected pumping capacities, thus causing a rather wide dispersion of the plotted points. The pumping capacities required in terms of inches per day give rather large values, but it must be remembered that these capacities represent peak rates that are needed for only a few hours at a time.

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[9] *Transactions*, Am. Soc. C. E., Vol. 106 (1941), Fig. 21, p. 1504.

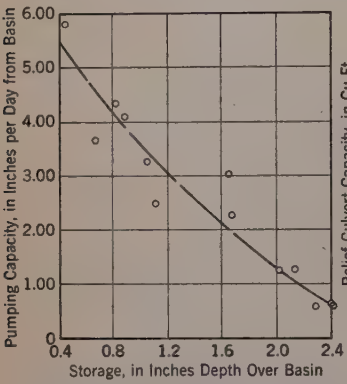

Fig. 5.—Relation Between Capacity and Flood-Plain Storage for Various Pumping Station Designs

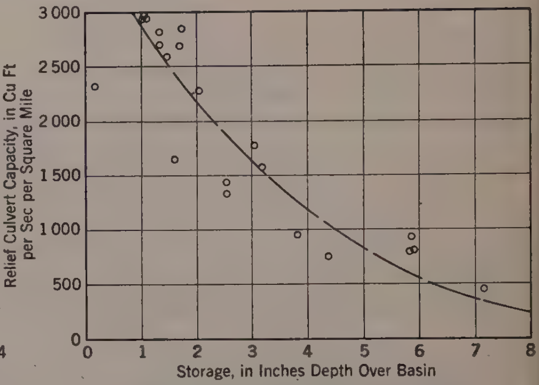

Fig. 6.—Relation Between Capacity and Flood-Plain Storage for Various Relief-Culvert Designs

The relations between relief-culvert capacity and flood-plain storage for a number of designs are shown in Fig. 6. In this case the inflow is that from the maximum probable flood. These relations are also greatly affected by channel and storage conditions in each individual basin.

It may appear to some that the basis of design for drainage structures presented herein is rather conservative, but it should be for the following reasons:

(*a*) Only a part of the present flood-plain storage is within the right of ways of the projects and hence is subject to reduction by filling in for future development;

(*b*) Future development may increase incremental damage resulting from ponding; and

(*c*) Future development may cause an increase in the volume and concentration of runoff.

Acknowledgments

Flood-control work on the Susquehanna River in Pennsylvania has been conducted by the U. S. Engineer Office, at Baltimore, Md., under the following district engineers: Col. E. J. Dent and Col. F. C. Boggs, Members, Am. Soc. C. E., and Col. W. A. Johnson and Lt.-Col. H. L. Robb. E. W. Digges, Assoc. M. Am. Soc. C. E., is chief of the flood-control division. Acknowledgment for contributions to the studies described herein and for assistance in the preparation of this paper is made to Mr. Kirpich, to E. H. Bourquard, Jun. Am. Soc. C. E., and to many other members of the Baltimore Engineer Office. The opinions expressed herein are those of the writer and do not necessarily reflect the policies of the U. S. Engineer Department.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

HYDRAULIC DESIGN OF DROP STRUCTURES FOR GULLY CONTROL

By B. T. Morris,[1] Jun. Am. Soc. C. E., and D. C. Johnson,[2] Assoc. M. Am. Soc. C. E.

Synopsis

In the stabilization of gullies, small overflow dams are used to retain silt and to control the stream grade. These dams are simple drop structures similar to those used in irrigation canals. In this paper the development of rules for the proportioning of such dams is described in terms of the hydraulic requirements for structure performance. The formulas included in the design rules are presented graphically for convenience in application. These rules are based on the accumulated experience of engineers in irrigation and soil conservation work and on the results of a series of laboratory test programs.

Design Problem

Small overflow dams, called drop structures, are installed in a gully to establish permanent control elevations below which an eroding stream cannot lower the channel floor. These dams control the stream grade, not only at the spillway crest itself, but also through the ponded reach upstream from the dam. Thus drop structures, placed at intervals along a gully, can stabilize it by changing its profile from a continuous steep gradient to a series of more gently sloping reaches separated by artificial spillways.

Although the construction of large numbers of drop structures for the stabilization of gullies in many different localities began with the operations of the Soil Conservation Service (SCS), U. S. Department of Agriculture, the development of designs for dams of this type started many years earlier with their application to the control of grades in irrigation canals. There is a marked similarity between dams that have been built in large gullies in the Pacific Southwest and canal drops (see Figs. 1 and 2).

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May, 1942.

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Records of the design and construction of canal drops are readily available in the literature of irrigation engineering in the United States and the British Empire. Engineers of the Soil Conservation Service have adapted design rules taken from this source to gully-control drop structures. The design rules presented in this paper have been developed from a typical canal-drop design through the analysis of the differences in the operating requirements of the two types of drop and through hydraulic laboratory tests of experimental drop structures.

Notation.—The following letter symbols, introduced in the paper, conform essentially to American Standard Letter Symbols for Hydraulics, prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1941:[3]

a = offset for weir notch ventilation (Fig. 3);
b = width:
    b_n = width of notch (Fig. 3);
    b_w = width of water surface upstream from notch;
C = coefficient:
    C_L = coefficient of apron length $= \frac{L}{\sqrt{h\, d_c}}$ (Eq. 2);
    C_x = coefficient of longitudinal-sill spacing $= \frac{x}{b_n}$ (Eq. 13*a*);
d = depth:
    d_c = critical depth of cross section;
    d_s = depth in upstream or downstream channel;
g = acceleration of gravity;
h = height; height of fall; h' = height of transverse end sill or depth of stilling pool;
L = length of the apron;
Q = discharge;
V = velocity:
    V_c = critical velocity of cross section;
    V_s = mean velocity in upstream or downstream channel;
x = spacing distance of longitudinal sills (Design B, Fig. 3);
x' = spacing distance of longitudinal sills (Design A, Fig. 3).

Gully-Control Problem vs. Canal-Drop Problem.—Both the fundamental purpose of drop-structure installation and the basic problem to be met in drop-structure design are common to canal and gully applications. The purpose is the control of stream grade; and the problem is the design of the spillway part of the structure. In each type of drop installation the permanence and the efficiency of the structure are controlled by the performance of the energy-dissipating and scour-preventing devices installed downstream from the dam proper. The efforts of the writers in the development of design rules for drop structures have been concentrated on the spillway problem. The type of energy-dissipating and scour-preventing device adopted by them, after

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[3] Publication pending.

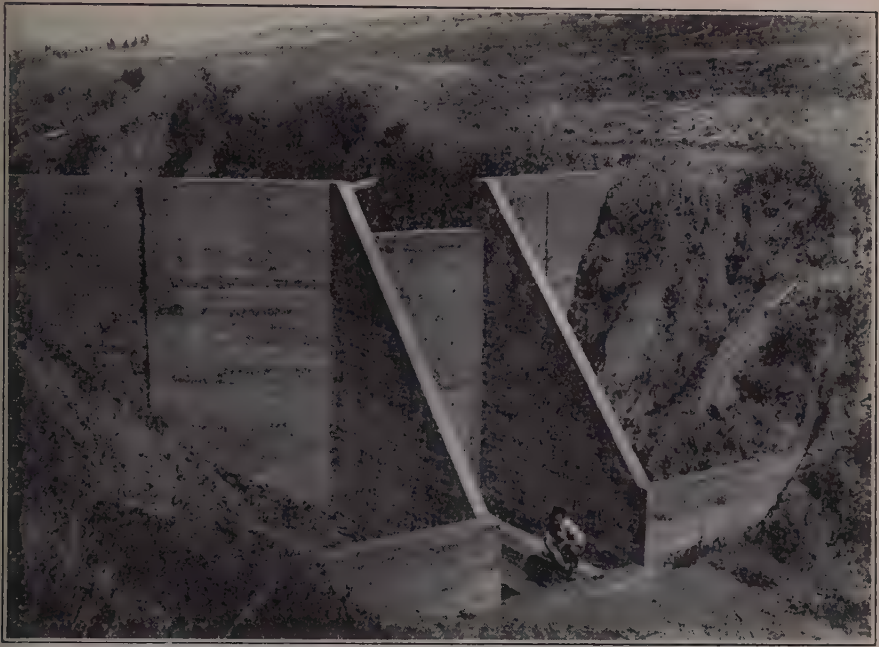

Fig. 1.—A Newly Constructed Drop on the Las Posas Project, Ventura County, Calif. (Before the Backfill Has Been Placed Between the Structure and the Banks)

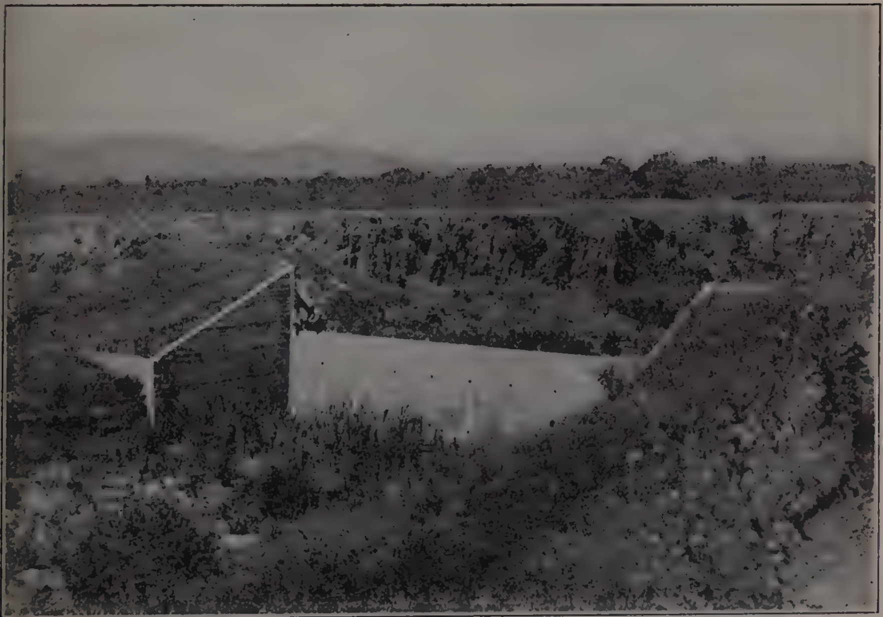

Fig. 2.—An Unusually Wide Structure in the Las Posas Barranca, Ventura County, Calif.

consideration of various types previously applied to small drop structures by others, is the rectangular apron with transverse end sill that is made an integral part of the dam and supporting walls. (This part of an overflow dam and spillway has been given several names descriptive of its effect on the flow—stilling pool, stilling basin, water cushion, spillway bucket, tumble bay, and cistern. Photographs of flow through the structure, as well as the analysis in this paper under "Criteria for Satisfactory Drop-Structure Performance" will demonstrate the inadequacy of each of these terms taken literally. Needing a term to describe the function of this part of the structure and being unable to provide a satisfactory name, the writers, on occasion, will use the term "stilling pool." Suggestions of better names will be welcomed.) The simplicity of layout and construction of this type of drop structure, as well as the available knowledge of its performance, was recognized in its selection for standard design. Descriptions of its application to irrigation canals have been presented elsewhere by B. A. Etcheverry,[4] M. Am. Soc. C. E.

Differences between the characteristics of flow in irrigation canals and in natural gullies led to the selection of the apron with end sill in preference to the simple apron or any other device which depends on the tailwater stage for the establishment of a hydraulic jump immediately below the dam. The steep grades, irregular runoff rates, varying silt loads, and uncertain roughnesses of gully channels make the elevations of tailwater surface and stream bottom very difficult to predict and, at best, too unreliable for use in controlling the performance of the structure. Although there are many uncertainties involved in the prediction of equilibrium slopes and stable grades in earthen irrigation channels, the controlled rates of flow, mild slopes, and low velocities make stage determinations in canals much more reliable than those in gully channels. To make matters more difficult, in natural gullies grades are such that streams almost always flow near critical depth and often flow more shallowly. Under the latter circumstance the energy-dissipating and scour-preventing devices of drop structures must be independent of tailwater stage. The apron and sill combination is well suited to this last requirement.

The drop structure resulting from the combination of a straight breast wall dam and a rectangular apron with end sill is shown in Fig. 3. The important variables in determining the proportions of the stilling pool are: The height of fall, h; the length of the apron, L; the height of the end sill (or depth of the pool), h'; and the discharge Q, indicated by the critical depth for the weir notch

$$d_c = \left(\frac{Q}{b_n \sqrt{g}}\right)^{\frac{2}{3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which b_n is the width of the notch. (In Fig. 3, Design A provides nappe ventilation through the offset of the side-walls from the edge of the notch as well as through the lateral contraction of the flow at the crest; Design B provides nappe ventilation through flow contraction alone.) Professor Etcheverry has presented a rational formula,[5] relating these variables, that may be written, in terms of critical depth, d_c:

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[4] "Irrigation Practice and Engineering," by B. A. Etcheverry, Vol. III, McGraw-Hill Book Co., Inc., New York, N. Y., 1916, Chapter VII.

[5] Ibid., p. 235.

$$L \doteq C_L \sqrt{h\, d_c} \qquad (2)$$

in which C_L, the coefficient of apron length, is variously determined at values from 3.1 to 4.5. The assumptions upon which Eq. 2 is based are: (1) That the nappe trajectory is a parabola with its vertex at the crest of the dam; and (2) that the falling water strikes the apron at a constant fraction of its length (from the crest).

In applying Eq. 2 to the determination of apron lengths in gully-control drop design, it was believed that a smaller value of the length coefficient might be used. Whereas, in the design of drop structures for canal service, it might be well to include a factor of safety in the value of the coefficient, C_L, in gully-control service, the factor of safety for the entire system of structures is included in the determination of the design discharge rate, so that the inclusion of a factor of safety in C_L would be a compounding of safety factors. The compounding of two or more safety factors does not lead to economical construction. Unless the cost of constructing gully-control drops is kept as low as possible, it is very difficult to justify their widespread use on farm land.

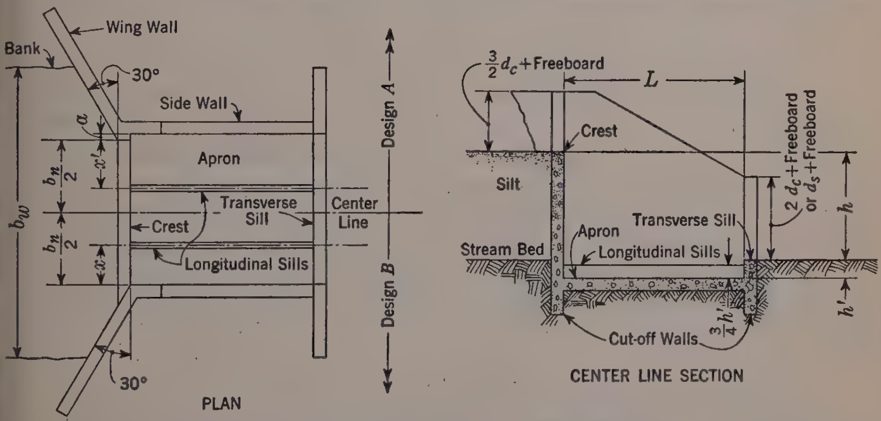

FIG. 3.—GULLY-CONTROL DROP STRUCTURE

The use of gully-control drops on agricultural land imposes an additional consideration over those taken into account in the design of the canal drop. Irrigation organizations and maintenance arrangements are such that preventive measures may be taken immediately wherever excess scour is noted at a drop structure. This factor is taken into account in the recommendation by many designers that riprap stream protection be placed downstream from drops and that this riprap be repaired and extended to suit conditions encountered. The drop design should not rely on maintenance by the farm operator for the safety of the structure. In gully-control service, the drop structure will receive maintenance that is no more likely to be regular than is its frequency of operation. For this reason, in the interest of the safety of the structure, there must be an increased emphasis on the dependability of the original design and construction of a drop for gully-control service. This factor of initial dependability is an economic factor fully as important as initial cost because it represents decrease in maintenance cost and decrease in the probability of loss through damage, failure, and the necessity of replacement.

Early Gully-Control Drop Construction.—The first drop structures built to designs prepared by the Pacific Southwest Region, SCS, had aprons designed with a length coefficient C_L of 2.5 and a sill height h' of $\frac{h}{8}$. The original intent of the designer was that the operation of these structures be observed and the design rules altered to suit observed deficiencies, if any. Some of these structures have been in place since 1936 and none have failed from stilling-pool inadequacy. However, observation of dams that have passed floods near design capacity has indicated an urgent need for improved control of the flow as it leaves the structure, as evidenced by serious scouring of the banks and channel bottom immediately downstream from the transverse sill and the end-walls.

As more and more structures were examined, it was realized that, within any reasonable length of time, sufficient data could not be obtained for the development of satisfactory design formulas. The improbability of the simultaneous presence of flood flows, competent observers, proper instruments, and access to structures was enough to discourage reliance on observation alone. It became clear, therefore, that some other means had to be used in developing the sound rules for design that would be necessary to the joint improvement of the economy and the dependability of the drop-structure method of gully control. Such a means was recognized in controlled experiments in the hydraulic laboratory.

HYDRAULIC LABORATORY TESTS OF EXPERIMENTAL DROP STRUCTURES

The problem of the hydraulic design of drop structures was referred to the hydraulic laboratory of the Soil Conservation Service at the California Institute of Technology, Pasadena, Calif., where a series of experiments was planned for the development of satisfactory designs and rational design rules.

The first experiments dealt with a structure of typical proportions of height of fall, width of crest, and depth of flow. Although it was not essential to the interpretation of the tests that a scale model be used, the experimental drop structure was designed as a one-eighth scale model of a laterally-contracted drop structure (see Fig. 3, Design A) having the dimensions:

Crest width, b_n, in feet	9.6
Fall height, h, in feet	8.5
Discharge, Q, in cubic feet per second	350
Critical depth for notch cross section, d_c, in feet	3.45
Channel width, upstream, b_w, in feet	14.7
Apron width, $b_n + 2a$, in feet	10.0

Expressed in appropriate dimensionless ratios, these drop-structure proportions are: $\frac{\text{fall height}}{\text{critical depth at the notch}} = \frac{h}{d_c} = 2.5$; $\frac{\text{critical depth}}{\text{notch width}} = \frac{d_c}{b_n} = 0.36$; and contraction ratio $= \frac{\text{stream width}}{\text{notch width}} = \frac{b_w}{b_n} = 1.5$.

Stilling pools equivalent to the following combinations of prototype dimensions (in feet) were tested in conjunction with this experimental drop at flow rates from 28% to 200% of the hypothetical design capacity:

Length	Depth
16	2
20	2
24	2
24	3

The downstream channel of the drop installation was represented by a model that had a trapezoidal cross section with 1 on 1 side slopes and a bottom width corresponding to 10.0 ft in the prototype.

The experimental drop-structure installation differed from the prototype it represented in that it consisted of only one half of the symmetrical drop and channel system. A sheet of heavy plate glass, placed at what would have been the center line of the complete structure, made observation possible without distorting the flow sufficiently to impair the similitude required for this type of hydraulic experiment.

Another compromise with nature made interpretation of the experiments difficult. To shorten the time required for tests and to simplify the test technique, solid channel walls and bottom were provided where field installations would furnish natural erodible materials. Thus, the experimental channel had fixed boundaries instead of a movable bed. In an installation of this kind there was no direct measure of the scouring power of the effluent stream. Therefore, other performance criteria had to be chosen that would aid in the selection of the best structure proportions.

Criteria for Satisfactory Drop-Structure Performance

In order to judge the excellence of drop-structure performance, the necessary qualities of a good drop structure must be established in terms of drop-structure use.

First, the structure must drop the water within its own confines and discharge it downstream in such a way as to cause a minimum of locally intensified erosion; and

Second, in order that the structure may perform its first function continuously, it must discharge the stream in such a manner that the flow will not undermine the structure itself.

These two requirements will be recognized by hydraulic engineers as those governing all spillway construction.[6] One difference is important: In soil conservation work, the order of importance of the requirements is the reverse of that found in most spillway work because the reduction of erosion is the original purpose for which the structure is erected. In other types of dam construction, the first rule is generally of secondary importance and there are even some cases in which this consideration has been neglected entirely, although the writers find it difficult to conceive of an installation where there is not some value in saving the channel from gross enlargement.

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[6] "Low Dams" (handbook), National Resources Committee, Washington, D. C., 1938, p. 108.

Before these rules can be applied directly to the interpretation of the results of experiments, they must be redefined and restated in terms of the behavior of the flow through the structure. In order that this flow may conform to the rules for structure performance, the kinetic energy of the falling water must be "dissipated" through its conversion to turbulence energy in the eddy motion of the "stilling pool" and this turbulence energy must be so distributed in the flow (prior to its complete decay through conversion to heat energy by viscous forces) that it will have a minimum of sediment-transporting power and thus a minimum of scouring power. Furthermore, the flow over the end sill must produce a movement of sediment along the channel toward the end sill, rather than away from it, so that undermining may be prevented.

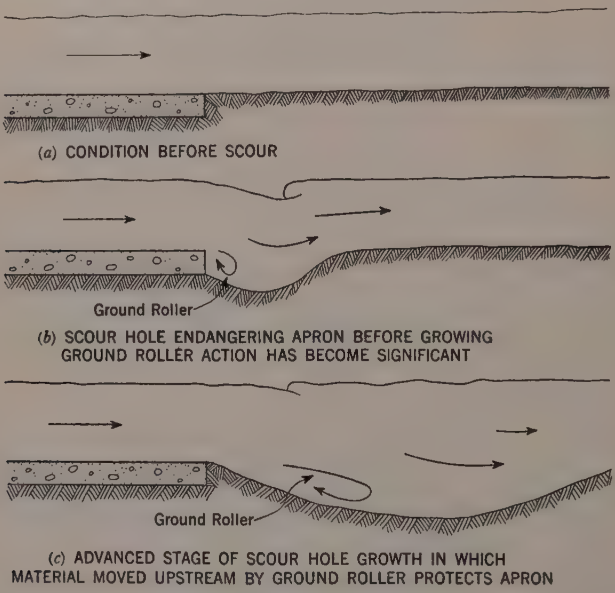
(a) CONDITION BEFORE SCOUR
Ground Roller
(b) SCOUR HOLE ENDANGERING APRON BEFORE GROWING GROUND ROLLER ACTION HAS BECOME SIGNIFICANT
Ground Roller
(c) ADVANCED STAGE OF SCOUR HOLE GROWTH IN WHICH MATERIAL MOVED UPSTREAM BY GROUND ROLLER PROTECTS APRON

FIG. 4.—SCOUR HOLE FORMATION

In order that the flow through a drop structure may satisfy this second set of rules, certain detailed requirements must be met. At the downstream end of the structure the larger eddies and stronger velocity filaments of the stream should not be directed toward the bank. Instead, the flow in this danger zone should be made as quiet and low in eroding power as possible. The reasons for these statements become clear when the equilibrium of scour and deposition at a gully bank is considered. The equilibrium of scour and deposition in the various parts of a natural gully cross section has been treated by N. A. Christensen,[7] Assoc. M. Am. Soc. C. E.

So far as erosion by the stream may be concerned, such equilibrium is often imaginary, for the stream itself cannot deposit material on a bank whose slope is equal to, or greater than, the angle of repose of the inundated material. The bank outline may remain fixed only if sufficient material is added through sliding or flow down the bank to match that withdrawn by the stream. On the basis of these considerations it may be seen that the banks below the structure will be eroded by the stream until this equilibrium can be reached. Therefore:

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[7] "Some Aspects of Gully Development, Classification and Control," by N. A. Christensen, thesis presented to California Institute of Technology in 1938 in partial fulfilment of the requirement for the degree of Doctor of Philosophy.

> Every effort that can be made to reduce the lateral attacking power of the stream will reduce the extent of channel widening.

Just as the components of high transporting power must be kept away from the stream banks at the exit from the stilling pool, so must they be kept away from the stream bottom. The designer and constructor of a drop structure are tempted, at first, to try to decrease the danger of scour by making an "easy" transition from the end sill or apron to the gully floor. Unfortunately, a smooth plane extension of the natural sediment bottom from the masonry of the structure (see Fig. 4) is not always stable under the flow of the stream. If such a surface were maintained by the equilibrium of scour and deposition of the stream, the stability of any local part of the surface (say, at the downstream edge of the structure) would be sensitive to random or accidental fluctuations in the transporting power of the stream. Such a fluctuation would produce a small disturbance in the bottom composed of a pit and a dune of excavated or disturbed material. This disturbance in the bottom will itself give rise to further local increase in the transporting power of the stream. Thus, the requisites for instability are present in the phenomenon, and a scour hole is certain to develop. The rate of growth of this scour hole is controlled, at first, by the balance of the strength of the initial disturbance and the stability of the sediment particles and, later after the scour hole has attained considerable depth, by the relative depths of the scour hole and the stream itself.

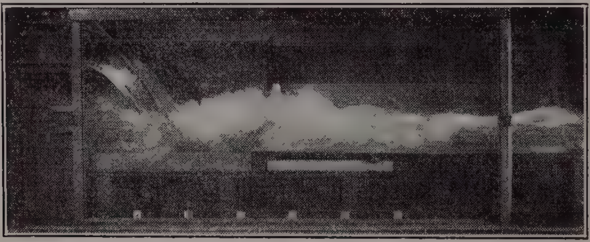
Recognizing the futility of scour hole prevention as a means of protecting the structure from caving, the designer must see to it that the hole that is formed does not endanger the structure. This he may do by forcing it to be developed far enough downstream from the end sill of the structure that the part adjacent to the structure will be too shallow to be dangerous with normal cutoff wall provision. After the first development of a scour hole downstream from the structure, the scour process itself tends to place the deepest part of the hole farther and farther downstream. The discontinuity of flow lines formed at the downstream end of the structure, when scour lowers the stream-bed level, encourages separation in the flow.

This separation is evident in the form of a "ground roller" whose top elements move downstream and whose bottom elements move upstream. Until this ground roller is formed, sediment immediately downstream from the structure can be removed by the stream but cannot be replaced because the motion of the water and the entrained sediment is downstream. After the ground roller has been formed, the upstream flow adjacent to the bottom can bring sediment from the downstream parts of the scour hole to replace that removed through the roller. From this understanding of the behavior of the scour hole in the vicinity of the drop structure can be deduced a rule for the determination of desirable flow conditions at the end of the structure:

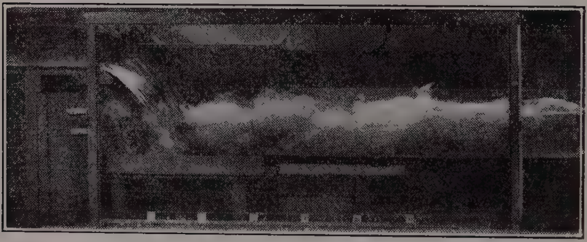
> The flow must be such as to develop a protective ground roller before the end of the structure has been laid bare to a dangerous depth.

(The importance of the "ground roller" to scour control has been emphasized many times in the literature of spillway design.)[8]

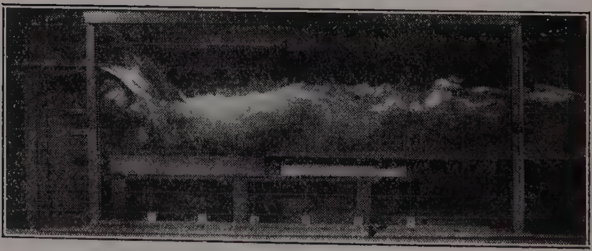
In order that the effluent stream shall have the minimum sediment-transporting power economically obtainable, the over-all dissipation effectiveness of the stilling pool should be a maximum and the excess energy in the effluent stream, a minimum.



(*a*) Tailwater Depth Less than Critical



(*b*) Jump Forming Immediately Downstream



(*c*) Stilling Pool Operating Submerged

FIG. 5.—PERFORMANCE OF AN EXPERIMENTAL DROP STRUCTURE AT THREE TAILWATER STAGES

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[8] "Stauraumverlandung und Kolkabwehr," by A. Schoklitsch, Julius Springer, Vienna, 1935, p. 85; "Low Dams" (handbook), National Resources Committee, Washington, D. C., 1938, p. 106; "The Causes and Prevention of Bed Erosion," by Arthur Douglas Deane Butcher and John Dekeyne Atkinson, *Minutes of Proceedings*, Inst. C. E., Vol. 235 (1932–33), pp. 175–222 and discussion; and "Dissipation of Energy below Falls," by C. C. Inglis and D. V. Joglekar, Bombay, P.W.D., *Technical Paper No. 44*, Bombay, 1933.

For the selection of the best structure from the laboratory experiments, the following forms of the drop-structure performance criteria were used:

(1) The kinetic energy of the effluent stream, as measured by the excess of local velocity over the velocity for absolute minimum specific energy, shall be a minimum (absolute minimum energy is obtained in flow at critical depth);

(2) The flow in the vicinity of the banks of the gully shall be as nearly parallel to the banks as possible and shall have a minimum of eddy motion; and

(3) A large ground roller shall be produced in the flow over the end sill of the stilling pool before any scour has occurred.

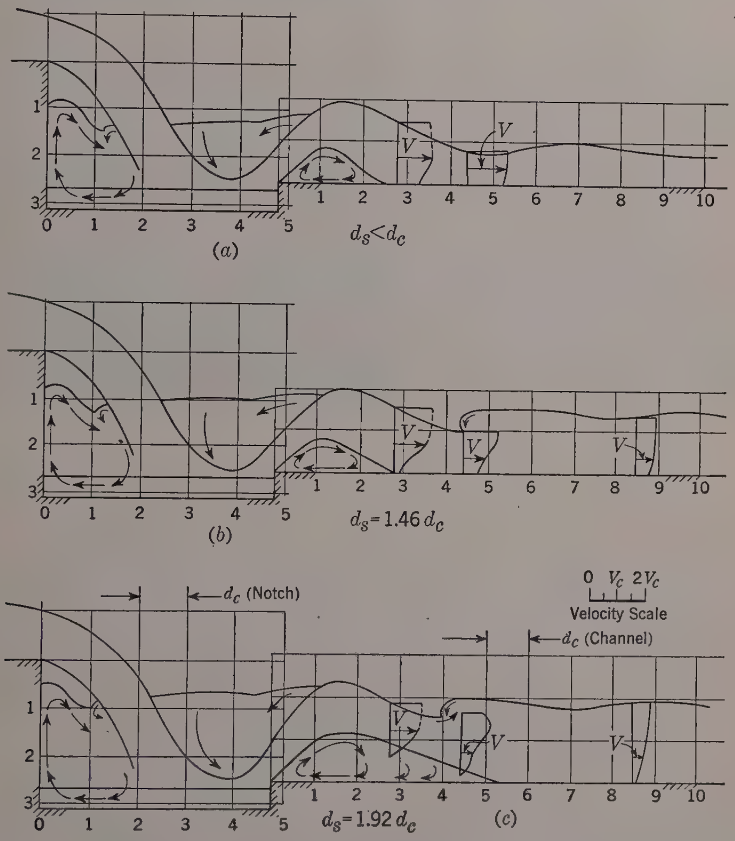
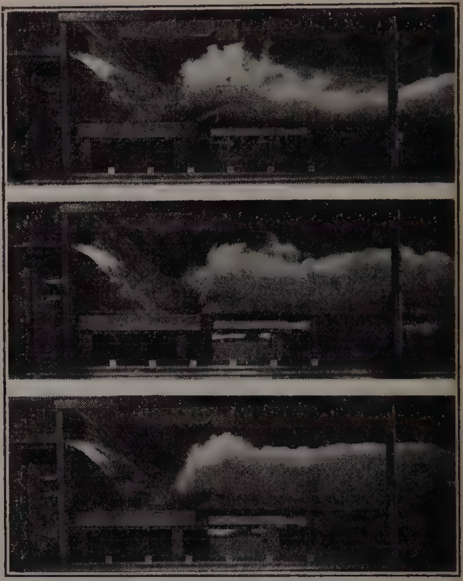


FIG. 6.—DIMENSIONLESS PLOTS OF FLOW PATTERNS AT THE CENTER LINE OF THE EXPERIMENTAL DROP STRUCTURE

RESULTS OF THE FIRST SERIES OF EXPERIMENTS

The first series of experiments with drop structures of varying stilling-pool design furnished considerable information in regard to the selection of apron lengths and sill heights, and led to the development of a new device for the reduction of bank scour.

Although photographs and flow and velocity measurement records were made of more than one hundred combinations of apron length and sill height, stream discharge and tailwater depth, it will be necessary to present only a few of them to demonstrate the type of drop-structure performance obtained.



A.—Tailwater Depth Less than Critical

B.—Jump Forming Short Distance Downstream

C.—Stilling Pool Slightly Submerged

(*a*) Twice Normal Discharge ($Q = 700$ Cu Ft per Sec; and $L = 16$ Ft)

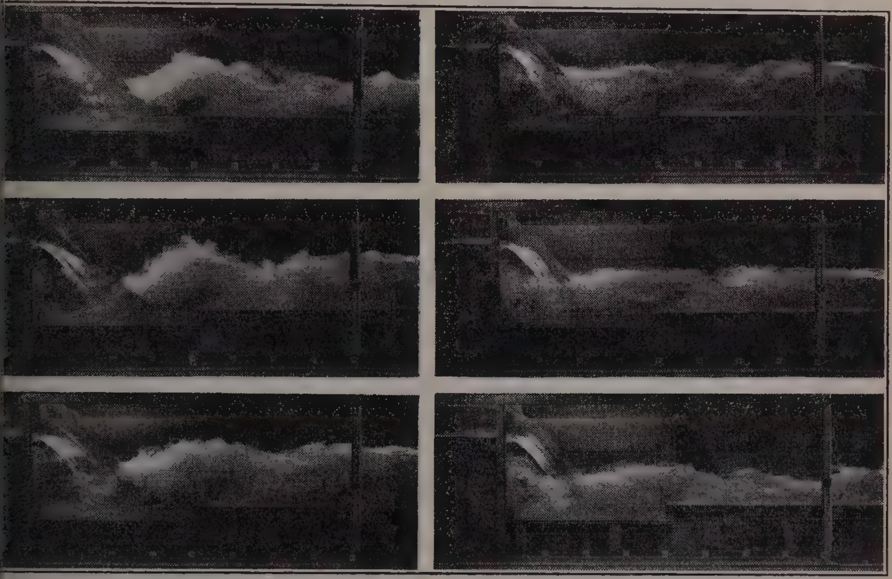
Fig. 7.—Performance of the Experi-

Fig. 5 illustrates the performance of the experimental drop structure at the "design rate of flow" ($Q = 350$ cu ft per sec) with the stilling pool ($L = 16$ ft, and $h' = 2$ ft) that proved to be the best according to the foregoing standards. Measurements of velocity, as well as observations of the general characteristics of the flow, indicate that this pool was the best that was tried at any of the tailwater stages shown. Although the quality of the effluent stream is affected by the degree of submergence, the same stilling pool affords the best performance at any depth in the downstream channel.

Measurements in the downstream channel gave maximum values of the ratio of local velocity to critical velocity of 1.6, 1.2, and 1.1 for the conditions shown in Figs. 5(*a*), 5(*b*), and 5(*c*), respectively. These maximum local velocities occurred well above the stream bottom, away from the sides, and about one pool-length downstream from the end of the structure (see velocity profiles in Fig. 6).

The ground roller required for the protection of the structure from undermining is present even before the excavation of a scour hole might begin. The end sill of the stilling pool has performed a double task in improving the energy dissipation in the pool and in deflecting the departing stream upward sufficiently to insure the development of a ground roller.

When the first tests were made with this experimental drop structure, a large part of the flow leaving the stilling pool was directed against the bank with a high velocity. To reduce the bank scour that would accompany such conditions, flow-straightening longitudinal sills (see Fig. 3) were placed on the apron at the third points of its width. Only one of these sills may be seen in Fig. 5 because only one is needed in the half model.



wice Normal Discharge (Q = 700 Cu Ft per Sec) with Stilling Pool Lengthened (L = 20 Ft)

(*c*) Original Discharge (Q = 350 Cu Ft per Sec) with Stilling Pool Lengthened (L = 20 Ft)

MENTAL DROP STRUCTURE (h' = 2 FT)

To emphasize the fact that material has not been wasted in constructing stilling pools of the size shown in Fig. 5, the photographs of Fig. 7(*a*), which indicate the performance of the same structure under a prototype flow of 700 cu ft per sec, are shown. It is immediately apparent that the apron is too short for either satisfactory energy dissipation or proper development of the ground roller. Unless the stream bed were otherwise protected, such a flow would excavate a scour hole so close to the end sill and so deep as to endanger the stability of the structure itself.

The photographs in Fig. 7(*b*) indicate the size of the stilling pool necessary to accommodate the flow of 700 cu ft per sec just described. Velocity measurements have shown that the energy dissipation performance of this structure corresponds exactly to that of the first structure at 350 cu ft per sec. The fact that both pools have the same depth may be taken as an indication that the pool designed for the 350 cu ft per sec flow might be made a little shallower.

The photographs of Fig. 7(*c*) are presented to indicate the nature of flow in stilling pools that are wastefully long. Here the flow of 350 cu ft per sec has been handled in the stilling pool designed for 700 cu ft per sec. The slight excess of depth is not at once apparent, but the section of parallel flow between the impingement zone and the curving flow at the end sill is a direct indication of the wasted length. Here again velocity measurements have shown energy dissipation performance equivalent to that shown in Fig. 5.

Design rules based on the aforementioned drop-structure performance were introduced in the engineering standards of the Pacific Southwest Region. Within a year and a half, sufficient value had been attached to them by experience in their use, so that requests were made to the regional engineer and to the laboratory for data that would allow the extension of the design rules to structures of other proportions than those used in this first series of tests. Examination of Fig. 5 and Fig. 7(*b*) will indicate that the experimental data were restricted to fairly narrow falls, with height-to-flow-depth ratios, $\frac{h}{d_c}$, between 2.5 and 1.5. Such drop structures were certainly typical of those encountered in the field practice of the Service, but there were also many drops of more extreme proportions.

As a means of extending the application of this type of laboratory data to design, a second program of cooperative research and design development was outlined.

THE SECOND SERIES OF EXPERIMENTS

Because experience in the first test program had indicated techniques to be used and the possibilities to be encountered in further testing, it was possible to plan the new test program in some detail and to design apparatus in advance. New, larger experimental drops were constructed in a testing flume of larger capacity than had been available before.

The new experimental drop (Fig. 8) was designed for operation at discharge rates from 0.5 cu ft per sec to 5.0 cu ft per sec for fall heights of 1.74 and 0.87 ft. Since the "half-model" technique was again used to facilitate examination of the flow at the center line of the structure, the 1.5-ft width of the supply flume corresponded to an approach channel width of 3.0 ft in a complete structure. Although the first arrangement of the experimental drop produced a condition geometrically similar to that used in the previous series of tests, this drop was not considered to be a model of any particular prototype structure. To emphasize the general applicability of the test results, all measurements were expressed as dimensionless ratios.

Rapidity of measurement and adjustment of the experimental variables and ease of visual and photographic observation were given consideration in the design of the experimental drop installation. The flume in which the new installation was made is of the closed circuit type and is equipped with a remote-control variable-speed pump and a pair of venturi meters. With this equipment the time required to change and redetermine the rate of flow through the experimental drop is very short. Timber and plywood construction made the alteration of the proportions of the drop itself a simple task.

The entire working section of the experimental drop installation was placed high enough above the ground to permit horizontal photography at convenient tripod and camera heights. The several sheets of heavy plate glass making up the window side of the installation were butt-joined with a transparent plastic cement that acted both as a water stop and as a structural "cushion" between the imperfectly cut edges of the individual panes. The top edge of the glass was supported laterally by removable crossties or by removable outside braces, depending on whether photographs were to be made from the side or from the top, respectively. The vertical load capacity of the glass sheets was supplemented by means of pipe stanchions which were kept in place except when photographs were to be made from the side.

A reference grid was established using hard-drawn aluminum wires that were spring-loaded to prevent sag from temperature change. The thrust of the spring anchorage was transmitted through the glass as an added safeguard against the opening of the butt joints.

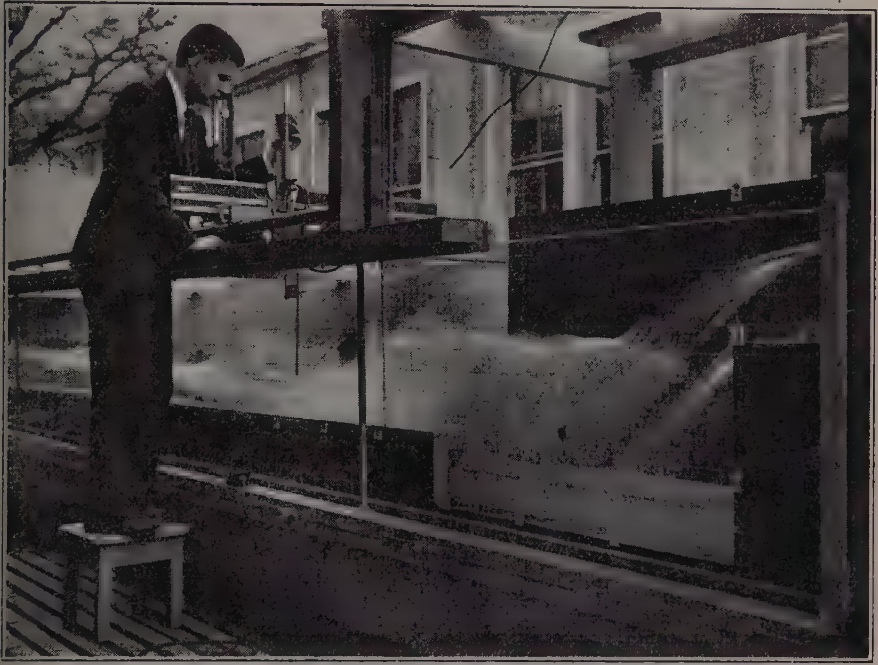


FIG. 8.—EXPERIMENTAL DROP STRUCTURE WITH PITOT TUBE AND TRAVERSING EQUIPMENT IN USE

A final precaution in the design of the drop installation, taken to insure undistorted photography, was the arrangement of space so that the cameras might be set 10 to 15 ft away from the glass.

Point-gage depth measurements and pitot-tube velocity measurements were facilitated by the provision of a wheeled carriage supporting traversing equipment and running on pipe rails. One of these pipe rails was connected to the domestic water-supply lines in order to provide pressure for flushing the pitot-tube and manometer velocity-measuring system. The carriage was made heavy and stiff to insure dependability of the reference planes in measurement.

In making this provision, the designer unintentionally furnished the experimenters with a solid but movable working platform that was very convenient in making photographs from above the drop and channel system.

Most of the photographs were made with a 5-in. by 7-in. reflex camera and a studio-type 16 mm motion-picture camera. All still photographs were developed and examined in negative form before proceeding with successive stages of the experiment. The care taken to insure good quality in the photographic work was thoroughly justified by the fact that most of the final conclusions of the study were based on measurements of the flow outlines as recorded in the photographs.

Sketches and velocity-distribution records were prepared only in the extent that they were necessary to the interpretation of the photographic record of the experiments. The decision to conduct the experiments under such a policy was made because of the great contrast between the time required for the two types of data recording and the recognized brevity of the time allowed for active testing. Only in this way was it possible to complete so widespread an investigation in nine weeks of active testing.

Selected parts of the motion-picture record of the experiments have been combined into an educational film which has been used in the instruction of field technicians in the Pacific Southwest Region. This film is considered to be a valuable auxiliary to the written report of the experiment.

RESULTS OF THE SECOND SERIES OF EXPERIMENTS

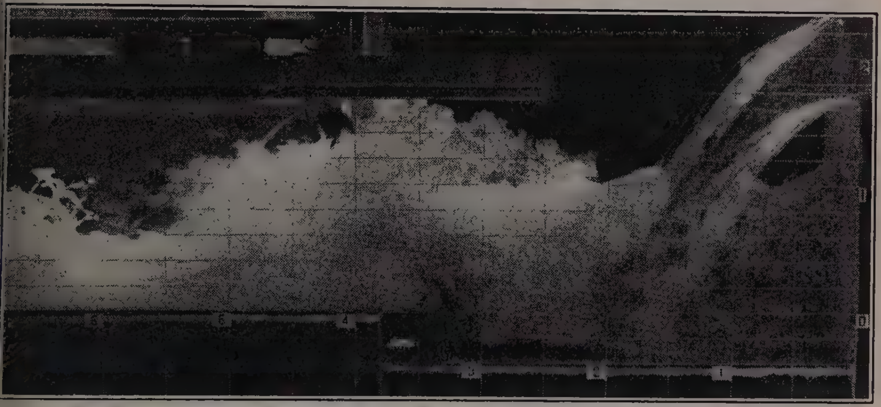
The first encouraging result of the second series of experiments was the duplication, through experiment, by different individuals with equipment of a different scale, of the results obtained in the first series of laboratory tests. Fig. 9(*a*) may be compared with Fig. 5 to demonstrate this point.

When the results of the first series were discussed, no photographs were presented that might show the purpose and result of the installation of the longitudinal sills on the apron. Fig. 9(*b*), taken from the second series, may be contrasted with Fig. 5 and Fig. 9(*a*) to show the performance of the structure without the longitudinal sills. Since the primary evidence of the improvement accomplished by the use of the longitudinal sills is the shifting of the high "plume" of rapidly moving water and spray from the bank to the center of the stream, these photographs do not show the difference as well as does Fig. 10, which demonstrates the performance with and without the longitudinal sill, respectively. Although the improvement in the flow conditions near the bank was sufficient justification for the use of longitudinal sills, it has been possible to detect an improvement in the over-all energy dissipation effectiveness of the stilling pool, as well. This improvement was noted in velocity measurements and in the decrease of the distance downstream to the beginning of the hydraulic jump for a given tailwater stage.

The tests of this second series covered a wide range of structures, from low drops with thick flows over their crests to high falls with thin sheets of water passing over them. In terms of the ratio of fall height to critical depth (calculated for the notch), $\frac{h}{d_c}$, the height range of the drops extended from 1.0 to 15.

Fig. 11 shows how stilling pools of appropriate length and depth, for falls of many different heights, were used to give performance equivalent to that

(*a*) **With Longitudinal Sill**



(*b*) **Without Longitudinal Sill**

FIG. 9.—PERFORMANCE OF THE SECOND EXPERIMENTAL DROP $\left(\frac{h}{d_c} = 2.5; \frac{L}{\sqrt{h\, d_c}} = 3.0; \text{ AND } \frac{h'}{d_c} = 0.60 \right)$

already described in Fig. 5. As in that experiment, the maximum local velocities were present in the flow well downstream from the structure and away from the banks and the bed of the stream. These velocities again ranged from 1.6 V_c for supercritical velocities in the downstream channel to 1.1 V_c for submerged operation with the tailwater stage at 2 d_c (V_c = critical velocity for the downstream channel).

To obtain the most efficient structures, the second test program included pools that had other proportions than those shown in Fig. 11. Inadequate “stilling” and dangerous distribution of the flow over the end sill characterized structures that were too small, such as the shallow pool of Fig. 12(*a*) and the short pool of Fig. 13(*a*).

The long stilling pool of Fig. 13(*b*) gave (like the long pool of Fig. 7(*c*)) performance that was not measurably better than that of the pools already described as satisfactory. Therefore, the use of long pools is regarded as wasteful

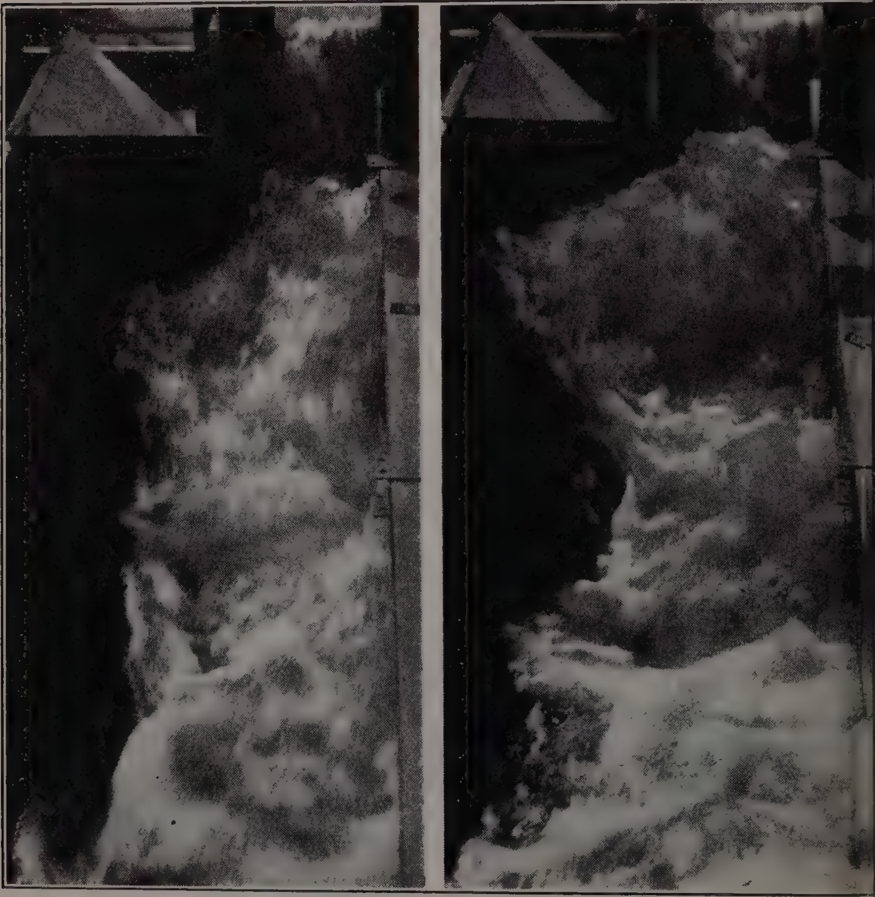

(*a*) With Sill (*b*) Without Sill

FIG. 10.—EFFECT OF THE LONGITUDINAL SILL ON DROP-STRUCTURE PERFORMANCE

and uneconomic. The performance of a stilling pool that is of the mos efficient length, but has been made four times as deep as the most efficien depth (see Fig. 12(*b*)), resembles the performance of the short structures o Figs 7(*a*) and 13(*a*). Although the energy-dissipation performance of the dee pool is fairly good, the direction of the flow over the end sill is such as to promote the formation of a deep scour hole immediately downstream from th

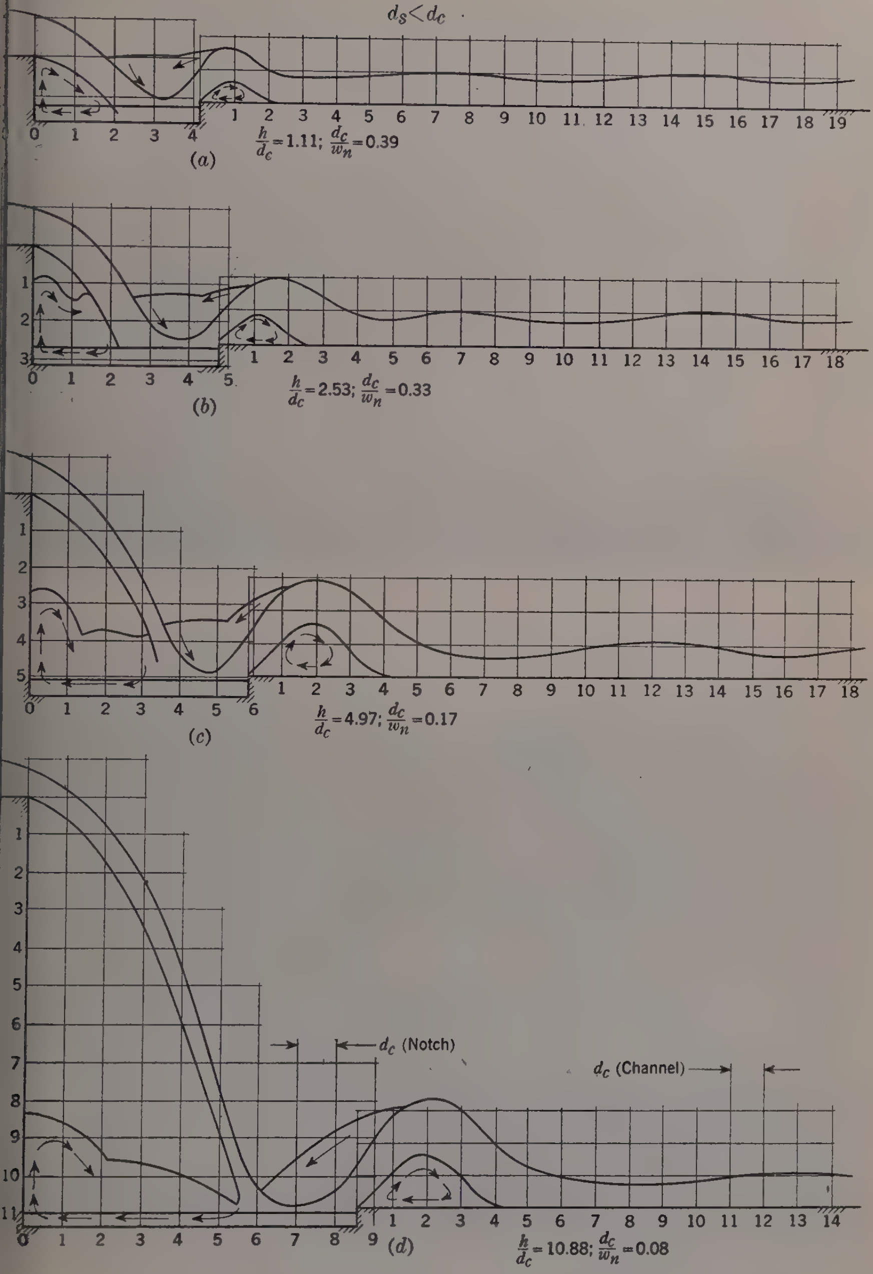
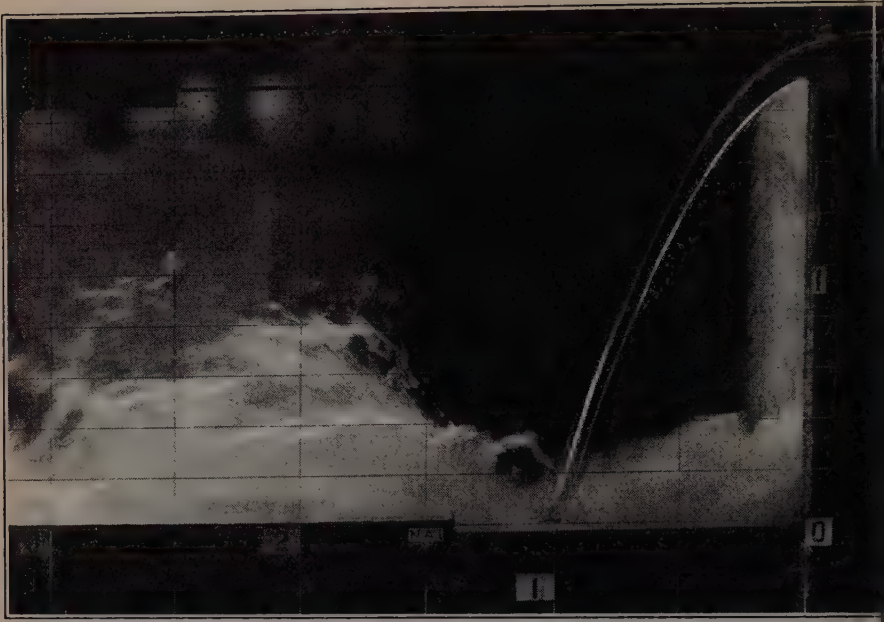
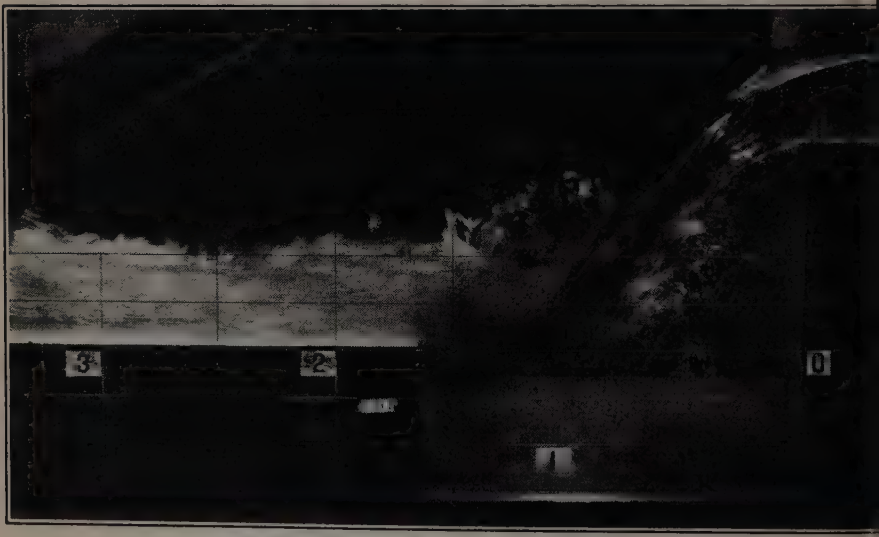


Fig. 11.—Flow Patterns for Drops of Various Relative Heights

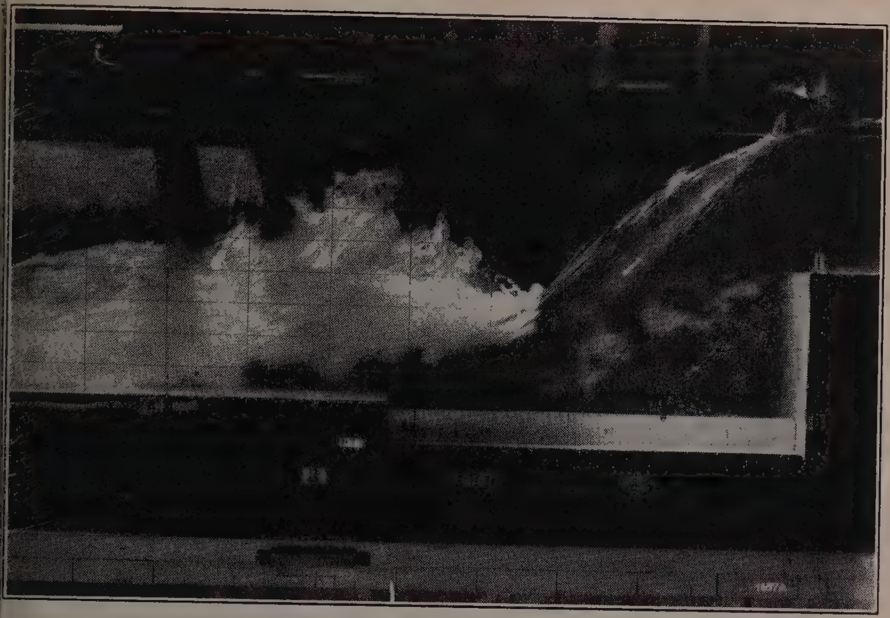


(*a*) Dangerously Shallow $\left(\frac{h}{d_c} = 10.9;\ C_L = 2.6;\ \text{and}\ \frac{h'}{d_c} = 0.31 \right)$

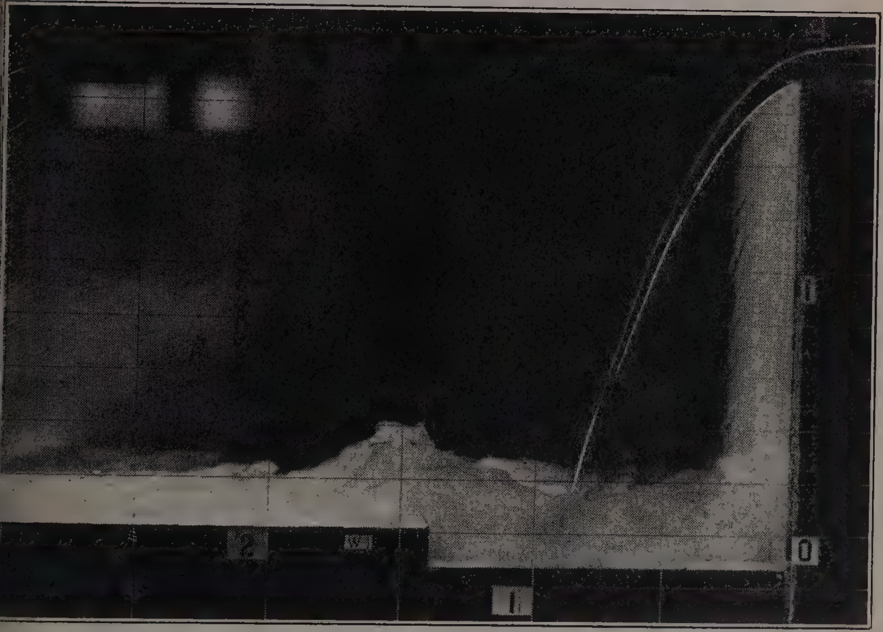


(*b*) Excessively Deep $\left(\frac{h}{d_c} = 2.5;\ C_L = 3.0;\ \text{and}\ \frac{h'}{d_c} = 1.95 \right)$

Fig. 12.—Effect of Depth of Stilling Pool



(a) Dangerously Short $\left(\frac{h}{d_c} = 1.0; \text{ and } \frac{h'}{d_c} = 0.30 \right)$



(b) Excessively Long $\left(\frac{h}{d_c} = 14.3; \text{ and } \frac{h'}{d_c} = 1.15 \right)$

FIG. 13.—EFFECT OF LENGTH OF STILLING POOL ($C_L = 3.0$)

structure. The writers did not conduct thorough tests of the possibilities of deep stilling pools in which this behavior would be eliminated by increase in the pool length but they believe that the improvement in energy dissipation that might possibly be gained in this manner would not warrant the great increase in construction cost necessary to obtain it.

Development of Design Formulas from Experimental Data

Although the experiment programs described herein were successful in developing efficient drop structures of various proportions, the development of these designs was not considered to complete the study. In order that structures of any proportions required in gully-control work may be designed with equal facility and dependability, it is necessary that rational equations or rules be written that will unite the information from individual designs into statements of general principles. From these studies it has been possible to develop, by rational methods, rules governing the length of aprons and the height of end sills and describing the location and proportions of the flow-straightening longitudinal sills.

1. Apron Length.—Eq. 2, which was already in use for the determination of apron length at the time the first test program was started, has a sound enough rational basis to make it a good starting point in developing a design formula for the expression of the results of the experiments. The height of fall and the critical depth for the notch are retained as the controlling variables.

The results of the tests shown in Figs. 5 and 7(*a*) suggested the use of Eq. 2 with a coefficient of $C_L = 3.0$. The establishment of this value of the coefficient, together with the experimental finding that a similar equation with a coefficient of $C_L = 2.0$ could be used to describe the trajectory of the water falling from the crest, suggests that the apron length requirement consists of two parts—(*a*) the distance required for the falling water to reach the apron and (*b*) the length of apron required to establish the energy-dissipating flow pattern.

For the time between the two series of tests, $C_L = 3.0$ in Eq. 2 was made the standard for apron length determinations. That the single value of the coefficient used in this equation should be too great for one extreme type of drop structure and too small for another was not surprising. The unsatisfactory performance of designs arrived at through its use has already been shown in Fig. 13.

As information was produced in the second series of tests, attempts were made to analyze the changes that occurred in the performance of structures of increasing or decreasing height. The experiments on high falls with thin flows were conducted first.

Soon it appeared that the distance between the point of impingement of the nappe and the end sill should not necessarily be either equal to or proportional to $\sqrt{h\,d_c}$. It seemed more likely that this distance should be related to the thickness of the impinging sheet of water. If this sheet has an original thickness proportional to d_c, then, neglecting the effects of friction and of mixing of air and water, it may be shown that the acceleration of gravity will make the thickness after falling a distance, h, proportional to $d_c \sqrt{\frac{d_c}{h}}$. Assuming that this thickness controls the remaining length of the apron, an equation for the coefficient, C_L, of Eq. 2, may be written:

$$C_L = 2.0 + C_1 \frac{d_c \sqrt{\frac{d_c}{h}}}{\sqrt{h\, d_c}} = 2.0 + C_1 \frac{d_c}{h} \quad \text{(3)}$$

This change in the coefficient proved too drastic (see Fig. 11, $\frac{h}{d_c} = 4.97$, $\frac{h}{d_c} = 10.88$) and, by cut-and-try methods, a new equation was determined which represented the data correctly:

$$C_L = 2.5 + 1.2 \frac{d_c}{h} \quad \text{(4)}$$

From the success of Eq. 4 it must be concluded that the effect of the thinning of the falling jet on the additional length required after impingement is not so severe as it was at first assumed to be.

The extension of the tests into the low fall, thick flow range (see Fig. 11, $\frac{h}{d_c} = 1.11$), showed that the foregoing adjustment is not sufficient to account for all of the added length requirements of this type of fall. Here the physical picture is different; additional length is now required before impingement. It is not to be expected, in the first place, that the parabolic trajectory equation, with its origin at the crest of the drop, should apply satisfactorily to the description of the flow near the crest itself. The finite thickness of the flow alone is sufficient to make the nappe fall above and downstream from the location thus predicted.

A second feature of the flow changes the pattern in low falls: The equilibrium of forces and momentum changes in the vicinity of the point of impingement requires that the water under the fall should stand above the tailwater stage established by downstream conditions. This water under the fall contributes to the support of the nappe even before the reduction of the fall height creates support from submergence alone (see Figs. 9(a), 12(b), and 13(a)).

The development of an analytical expression to adjust the coefficient of the apron-length equation for these effects has been too complicated a task for this study. Instead, an empirical term has been developed to express the influence of those features of the flow that will require longer aprons for low-drop structures:

$$C_L = 2.5 + 1.1 \frac{d_c}{h} + 0.7 \left(\frac{d_c}{h}\right)^3 \quad \text{(5)}$$

If this coefficient is used, the complete equation for the stilling-pool length may be written:

$$L = \left[2.5 + 1.1 \frac{d_c}{h} + 0.7 \left(\frac{d_c}{h}\right)^3\right] \sqrt{h\, d_c} \quad \text{(6a)}$$

or

$$\frac{L}{\sqrt{h\,d_c}} = 2.5 + 1.1\,\frac{d_c}{h} + 0.7\left(\frac{d_c}{h}\right)^3 \ldots\ldots\ldots\ldots\ldots (6b)$$

For the convenience of designers Eq. 5 has been plotted in its dimensionless form in Fig. 14. In developing Eqs. 2 to 6 it has been assumed that the dam crest constitutes a control section. If shooting flow in the upstream channel is not forced to jump before reaching the crest, these pool-length rules are not applicable to the design of the structure in question.

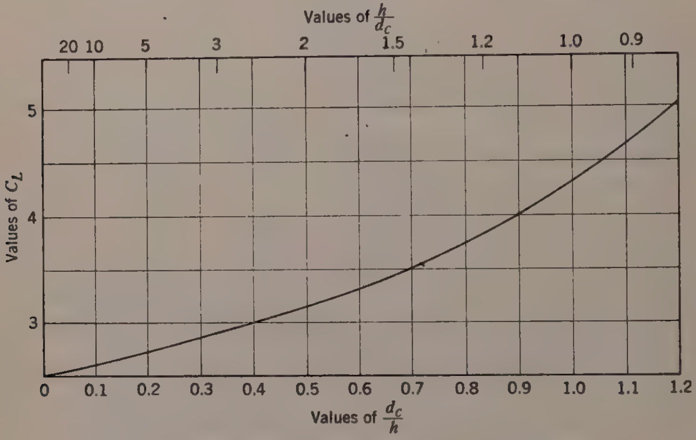


FIG. 14.—THE APRON LENGTH RULE (CURVE OF EQ. 6)

2. Height of Transverse End Sill.—The rules used for the determination of the end-sill height have been completely changed during the period of the laboratory work on drop structures. At the beginning of this period, the standard design of the Engineering Division of the Pacific Southwest Region called for a stilling pool whose depth was one eighth of the fall height, so that

$$h' = \frac{1}{8}\,h \ldots\ldots\ldots\ldots\ldots (7a)$$

After the tests shown in Fig. 5 and Fig. 7(*b*) were made, this rule was discarded in favor of

$$h' = 0.10\,L = 0.3\,\sqrt{h\,d_c} \ldots\ldots\ldots\ldots\ldots (7b)$$

At the conclusion of the second series of experiments the third rule was stated:

$$h' = \frac{1}{2}\,d_c \ldots\ldots\ldots\ldots\ldots (7c)$$

in which d_c is still the critical depth at the weir notch. The weakness of the second rule is apparent in Fig. 13 which has been cited as belonging to the same stage of design development.

This form of the sill-height rule (Eq. 7*c*) was not anticipated before the experiments were completed and was accepted, with some reluctance at first, as the only reasonable statement of the results of the experiments. Rather than attempt a complete analysis, the writers will state their conception of the factors that make this relationship possible.

The height of the end sill of the stilling pool necessary to develop energy-dissipating action is apparently determined by the depth and energy content of the flow in the pool.

Other tests conducted in the laboratory have demonstrated[9] that the loss of kinetic energy incident to the impingement of the nappe on the apron is a large quantity and that this loss increases rapidly with increasing fall height so as to offset, to an appreciable extent, but not completely, the increased energy of the higher falls. From these findings it can be stated that the flow away from the impingement zone still will be shallower for high falls than for low drops. Such shallow flows, of course, will have greater momentum than the deeper ones. It was desired to turn these shallower, swifter flows upward and over the end sill in such a manner as to insure the development of the energy-dissipating flow and the ground roller so essential to the protection of the structure. For this purpose the end sill must be proportionally higher than for the deeper slower currents from the impingement of lower falls.

The results of the experiments would indicate that the increase in relative sill height with increasing velocity and decreasing flow depth in the pool is such that the same depth of stilling pool, or height of end sill, relative to the flow depth at the crest is required for all fall heights.

3. Location of Longitudinal Sills.—Longitudinal sills were introduced as flow-straightening devices in drop-structure stilling pools for the first time during the first series of experiments. The size and shape of the sills were established during the first stages of the experiments and have remained unaltered so far as recommended designs may be concerned. Sills, three quarters of the end-sill height, are the most satisfactory. The width of these sills is determined by the materials used and structural considerations in general. For ordinary reinforced concrete construction (sills less than 3 ft high) a 6-in. width is adequate.

When the second series of experiments was planned, it was believed that the operation of the experimental drop at many flow stages at a constant width would produce data from which the possibility of other arrangements of the longitudinal sills than the third-point spacing originally used might be investigated. The experiments failed to satisfy this prediction. The third-point spacing furnished the best performance from beginning to end of the series of experiments originally planned.

From this result of the tests and from observation of the effects of the lateral contraction of the flow at the crest of the drop, it was concluded that the spacing of the longitudinal sills, like the lateral contraction of the nappe, was determined by the horizontal configuration of the contraction alone and was independent of the fall height and the flow depth. Accordingly, an auxiliary series of experiments was conducted in which the amount of the contraction of the flow at the crest of the drop was varied, whereas the fall height and the flow depth were held constant.

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[9] "Energy Loss at the Base of a Free Overfall," by Walter L. Moore, *Proceedings*, Am. Soc. C. E., November, 1941, p. 1697.

Fig. 15(*a*) shows the performance of the drop with the sill at the third point of the width of the hypothetical complete structure. The pronounced contraction of the flow and the presence of an offset for the ventilation of the nappe make this, the contraction used in all previous experiments, a very severe lateral effect. The distance between the edge of the nappe and the side-wall of the stilling pool may be considered to be a measure of this severity.

Figs. 15(*b*) through 15(*d*) show, continuously, the effect of the successive elimination of the causes of the contraction of the nappe. In each case the longitudinal sill has been placed in a position for maximum efficiency in flow straightening. In the case of parallel flow (Fig. 15(*d*)) this adjustment of the longitudinal sill has necessitated its complete removal since its ultimate location would be flush with the side-wall.

The importance of flow contraction at the drop-structure crest to the production of crosscurrents over the end sill was noted by Professor Etcheverry,[10] who recommended avoiding lateral contraction where possible. In gully-control work the shape of natural gully cross sections makes contraction difficult to avoid in any simple structure. The flow-straightening sills are the simplest and most economical devices that may be used to counteract contraction effects.

Although an investigation of nappe-ventilation provisions had not been anticipated in either series of drop-structure tests, the provision of ventilation by lateral offset or contraction, the two flow features governing the longitudinal-sill location, has become an inseparable part of the design problem.

It is possible to describe, with some confidence, the flow phenomena that govern the longitudinal-sill locations. As the contracted, or offset, flow passes from the crest of the drop to the stilling pool, the nappe fails to span the apron width uniformly. This lateral inequality becomes effective, at impingement, as a pair of lateral pressure gradients giving the flow over the end-sill components of velocity toward the banks of the gully. The function of the longitudinal sills is to prevent the development of such lateral components by separating the high-pressure and low-pressure regions of the impingement zone. It becomes clear, from this analysis, that the longitudinal sills must be placed near the third points when the pressures are changing over a large part of the apron width and nearer to the side when the region of pressure differences is small.

In the development of an equation for the spacing of the longitudinal sills as controlled by the notch contraction and the ventilation offset, it has proved more convenient to use an analogy with another flow contraction phenomenon than to attempt to follow step by step the process of producing the pressure gradients on the apron.

As the stream flow above the drop is contracted, it experiences a drawdown from the depth before contraction to a depth slightly less than the critical at the notch itself. The effect of curvature is to make this last depth depart from the calculated critical depth; but if this difference is neglected, the drawdown

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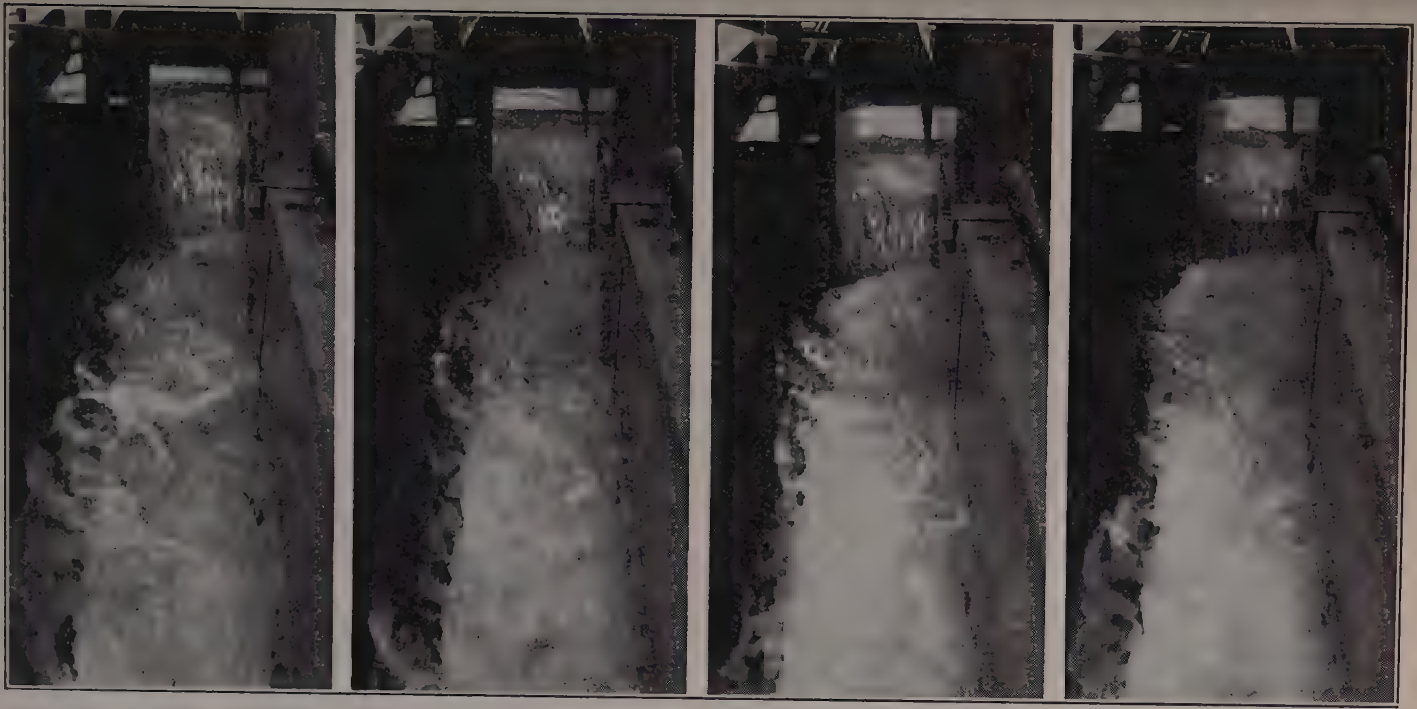
[10] "Irrigation Practice and Engineering," by B. A. Etcheverry, Vol. III, McGraw-Hill Book Co., Inc., New York, N. Y., 1916, pp. 238, 243.



(a) $\frac{b_w}{b_n} = 1.47$; $\frac{a}{b_n} = 0.031$; $\frac{x'}{b_n} = 0.32$ (b) $\frac{b_w}{b_n} = 1.38$; $\frac{a}{b_n} = 0$; $\frac{x}{b_n} = 0.21$ (c) $\frac{b_w}{b_n} = 1.11$; $\frac{a}{b_n} = 0$; $\frac{x}{b_n} = 0.12$ (d) $\frac{b_w}{b_n} = 1.00$; $\frac{a}{b_n} = 0$; $\frac{x}{b_n} = 0$

FIG. 15.—EXPERIMENTS ON THE CONTROL OF THE SPACING OF LONGITUDINAL SILLS BY LATERAL CONTRACTION

can be computed from the energy of a stream flowing at critical depth and the Bernoulli energy equation written for the section upstream from the contraction and the section at the notch.

The energy in the flow over the crest is

$$\frac{3}{2} d_c = d_s + \frac{V_s^2}{2g} \tag{8}$$

which is the energy upstream from the contraction.

From continuity requirements,

$$Q = V_n\, b_n\, d_n = V_c\, b_n\, d_c = V_s\, b_w\, d_s \tag{9a}$$

and

$$V_s = V_c \frac{b_n\, d_c}{b_w\, d_s} = \frac{b_n\, d_c}{b_w\, d_s} \sqrt{g\, d_c} \tag{9b}$$

Substituting in Eq. 8: $\frac{3}{2} d_c = d_s + \frac{1}{2} d_c \left(\frac{b_n}{b_w}\right)^2 \left(\frac{d_c}{d_s}\right)^2$; and

$$\frac{3}{2}\left(\frac{d_s}{d_c}\right)^2 = \left(\frac{d_s}{d_c}\right)^3 + \frac{1}{2}\left(\frac{b_n}{b_w}\right)^2 \tag{10}$$

Substituting the identity,

$$\frac{d_s - d_c}{d_c} + 1 = \frac{d_s}{d_c} \tag{11}$$

in Eq. 10 and transposing, the drawdown equation is obtained,

$$\left(\frac{d_s - d_c}{d_c}\right)^3 + \frac{3}{2}\left(\frac{d_s - d_c}{d_c}\right)^2 = \frac{1}{2}\,\frac{\left(\frac{b_w}{b_n}\right)^2 - 1}{\left(\frac{b_w}{b_n}\right)^2} \tag{12}$$

The tests devised for the study of sill location as a function of the notch contraction showed that no sills were required when there was no contraction and that increasing contraction moved the sills from their hypothetical position flush with the walls to positions near the third points of the apron width. Comparison of the experimental values of $\frac{x}{b_n}$ with the corresponding computed values of $\frac{d_s - d_c}{d_c}$ indicated that each had the same functional trend with changing values of the contraction ratio. From this observation it was concluded that the sill spacing, the departure of the nappe edge from the plane of the notch edge, and the drawdown were similar measures of the contraction of the flow at the crest. From the experiments shown in Fig. 15,

$$C_x = \frac{x}{b_n} = 0.60\,\frac{d_s - d_c}{d_c} \tag{13a}$$

and

$$\left(\frac{x}{b_n}\right)^3 + 0.90\left(\frac{x}{b_n}\right)^2 = 0.108\,\frac{\left(\frac{b_w}{b_n}\right)^2 - 1}{\left(\frac{b_w}{b_n}\right)^2} \tag{13b}$$

or,

$$C^3_x + 0.90\,C^2_x = 0.108\,\frac{\left(\frac{b_w}{b_n}\right)^2 - 1}{\left(\frac{b_w}{b_n}\right)^2} \ldots\ldots\ldots\ldots\ldots (14)$$

Since cubic equations of the type of Eq. 13*b* do not lend themselves to quick solution, a graph of Eq. 14 in its dimensionless form is used in design work.

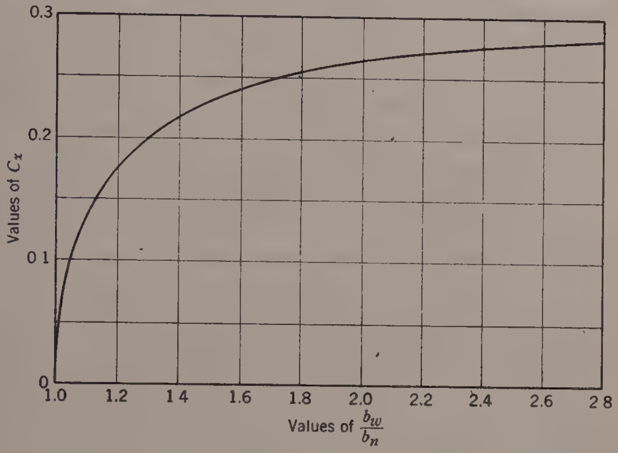


FIG. 16.—THE SILL-LOCATION RULE (EQ. 14)

This graph (Fig. 16) shows the relation between the sill spacing and the notch-contraction ratio.

The data on which Eq. 13*b* is based were determined from experimental drops with wing walls directed 30° upstream from the line of the crest. Although no experiments were made with walls of different orientation, sills located according to Eq. 13*b* should improve the flow conditions for almost any angle ordinarily met. The results of experiments with sill locations other than those recommended show that they still enabled the sills to improve flow conditions.

Eq. 13*b* gives the best sill location for drop structures that do not have an offset between the edge of the notch and the side-wall of the stilling pool. From what little information was obtained in tests it appears that the best results are produced if the distance between the sill and the edge of the notch, x, is increased by an amount equal to three times the notch offset, so that $x = C_x\, b_n$ and

$$x' = x + 3\,a \ldots\ldots\ldots\ldots\ldots (15)$$

The contraction of the flow not only makes the edge of the nappe depart from the plane of the notch edge, but also distorts the cross section of the nappe so that the flow is thickest near the center. The ventilation offset, for the purposes of these tests a small distance by comparison with the notch width, serves only to offset the nappe, the high pressure area on the apron, and the sills farther from the side-walls.

SUMMARY OF DESIGN RULES AND FORMULAS

Two general rules for all drop structures are:

1. The structure must drop the water within its confines and discharge it downstream in such a way as to cause a minimum of locally intensified erosion; and

2. The structure must discharge the stream in such a manner that flows less than, or equal to, the design discharge rate will not bring about the undermining of the structure itself.

Three specific rules for rectangular structures of the type treated in this paper are:

3. In order to provide efficient energy-dissipating action and to avoid waste of material, the design should provide an apron of a length derived by Eq. 6*a* (for graphical presentation of this formula see Fig. 14). Aprons shorter than this length are susceptible to undermining by scour at the end sill. Longer aprons do not give appreciable improvement in scour control. The application of Eq. 6*a* should be restricted to falls higher than $\frac{h}{d_c} \doteq 1.0$ because of the increasing importance of the empirical term, $0.7\left(\frac{d_c}{h}\right)^3$, to very low falls.

4. In order to provide sufficient depth without waste of material the end sill should have the height expressed by Eq. 7*c*. Lower end sills do not give as complete energy dissipation as sills of the recommended height. Higher sills add to the danger of undermining by scour unless they are used with longer aprons than would be provided by Eq. 6*a*. The amount of improvement in scour control, which may possibly be obtained through simultaneous increase in depth and length of stilling pools, is believed to be so small as not to justify the increased cost of construction.

5. If the drop is laterally contracted, longitudinal sills should be installed on the apron at a distance x' from each side of the weir notch as given by Eq. 15, in which x is determined from the solution of Eq. 13*b* (for graphical presentation of formula, see Fig. 16). These recommended spacings are based on the use of wing walls placed at angles of 30° with the axis of the dam, but should be applicable to most wing-wall orientations commonly used. Sill locations other than those recommended are less effective in flow straightening and scour control, but give better performance than does the complete omission of the sills.

The longitudinal sills should be made three quarters of the height of the end sill of the stilling pool. The width of the sills is determined by the strength of the material used for their construction. Since Eq. 15 is empirical and is based on measurements at $\frac{a}{b_n} = 0.031$, its application should be restricted to the range $0 < \frac{a}{b_n} < 0.031$.

It is recognized that the foregoing design rules are based on hydraulic considerations only. Therefore, they must be regarded as minimum requirements susceptible to increase on the basis of other design considerations.

The foregoing rules and formulas have been in use by the Engineering Division of the Pacific Southwest Region, SCS, since July, 1940.

Gully-Control Installations of Free-Overfall Drop Structures

The gully-control structures most nearly resembling those used in the experiments are constructed of reinforced concrete and are of substantial size. Structures similar to the one shown in Fig. 1 have been built for falls from 5 to 30 ft in height, from 8 to 75 ft in width, and from 50 to 5,000 cu ft per sec in discharge rate. To all sizes of structures of this type, the design rules developed in this study are directly applicable. Obviously, structural and economic considerations will limit the height of structures for which the free-overfall type of spillway is feasible; but the hydraulic principles involved in the designs given are independent of scale for flows more than a few inches deep.

Smaller structures are often more economically constructed of masonry using local stone. The same rules that govern the outline of the water passages in the reinforced concrete structure have been adapted to these. The downstream face of a masonry drop normally has an appreciable batter. Since these drops are ordinarily designed to operate at low ratios of height to depth, the nappe will not strike the face of the wall and, therefore, the starting point of the pool length has been considered to be the crest of fall. The space taken up by the sloping section of the breast wall is not considered to be important in the action of the stilling pool, except that the nappe should fall free of the face for its entire height.

Under special conditions of exposure and operation it is desirable to construct rectangular drop structures of logs or of timber. Here again, the rules developed for the reinforced concrete structure will govern the hydraulic proportions of the structure.

The writers do not expect that the limitations in the applicability of Eq. 6*a* to low falls will be encountered often in gully-control drop design, because of the possibility of the use of other types of structures in place of the free-overfall and rectangular stilling-pool type they have discussed.

Application of Drop-Structure Rules to Other Types of Hydraulic Structures

The principles used in developing the rules for the design of rectangular drop structures for gully control are sufficiently general for application to many problems. There is no great difference between the methods that have been used in insuring energy dissipation and in protecting the structure from undermining and those commonly used in the design of spillways for major engineering structures. Although structural and economic considerations require departure from the rectangular construction discussed herein, the flow patterns for major spillways and stilling pools are still such that kinetic energy is converted to turbulence in the pool and that the flow over the end sill produces a reverse ground roller that protects the end of the structure.

At the other end of the scale of spillway structures used in hydraulic engineering are temporary "checks" constructed of various types of inexpensive materials, such as boards, loose rock, and brush. Only the promise of inexpensive construction has kept this type of dam a part of engineering practice. Perhaps the application of some of the ideas presented herein may help in the identification and elimination of some of the characteristic weaknesses of this type of construction. Already experience has pointed out several types of failure: Undermining due to scour downstream, undermining due to percolation, lateral by-passing, and structural disintegration brought about by hydrostatic or hydrodynamic forces.

The first of these causes of check-dam failures has been analyzed in this study of the rectangular drop structure and it would appear that the thoughtful application of the design rules presented in this paper should prevent failures from this cause alone. The other three causes of failure may be regarded as expressions of rules limiting the use of cheaply-erected checks of various materials.

Conclusion

The writers have presented hydraulic design rules and formulas now in use by the Pacific Southwest Region, SCS, U. S. Department of Agriculture, and have described the experimental and analytical development of these rules at the laboratory of the Cooperative Research Project of the Soil Conservation Service and the California Institute of Technology.

The analyses presented in the development of design rules for drop structures may be regarded as statements of progress in a long-term investigation of the mechanics of energy dissipation and of localized scour. The writers realize that many problems remain to be studied, and hope that this paper will serve to indicate some of them. Conspicuous among items considered for future study are: (*a*) The importance of lateral contraction and divergence of the flow in the structure to energy dissipation performance; (*b*) stability of flow over the end sill; (*c*) energy dissipation at the end sill; (*d*) analysis of eddy-production in the lateral separation zone at the banks below the end wall of the structure; and (*e*) development of methods for reducing the scouring power of the eddy system of Item (*d*).

The particular studies reported in this paper were made through the cooperation of field technicians and a laboratory research group. It is anticipated that this valuable method of design data development will be continued.

Acknowledgments

The operations of the Soil Conservation Service in the Pacific Southwest Region are directed by Harry E. Reddick, Assoc. M. Am. Soc. C. E., Regional Conservator. Engineering activities in the Region are supervised by J. G. Bamesberger, Regional Engineer. The research work of the laboratory is under the direction of Robert T. Knapp, M. Am. Soc. C. E., for the California Institute of Technology, and Vito A. Vanoni, Assoc. M. Am. Soc. C. E., for the Soil Conservation Service.

Prior to August, 1938, the work of the laboratory in this particular field was directed by Mr. Christensen, to whom the writers are much in debt for the information developed in the first series of experiments. In the second series, the writers were assisted by Wilson B. Jones, Jun. Am. Soc. C. E., who made the experimental measurements and collected the data of that series.

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PAPERS

ANALYTICAL AND EXPERIMENTAL METHODS IN ENGINEERING SEISMOLOGY

BY M. A. BIOT,[1] ESQ.

SYNOPSIS

The arithmetical steps in the computation of the spectrum are extremely lengthy. A mechanical analyzer was developed by the author at Columbia University, in New York, N. Y., to avoid this numerical work.[2] Some of the writer's earlier work with the earthquake spectrum is reviewed briefly in this paper. It also stresses engineering applications, and presents some new results, in particular regarding the effect of the foundation. Sections 1 and 2 introduce the definition of earthquake spectrum and show the results obtained for various earthquakes with the mechanical analyzer. Section 3 is a treatment of the spectrum curves obtained with the analyzer in relation to some observed facts and to the problem of stress prediction in actual structures. Section 4 considers examples of structures with more than one degree of freedom and shows how the stresses may be computed by means of the effectiveness factor. (The expression "efficiency factor" instead of "effectiveness factor" was used in the previous paper.[2]) The danger of a phenomenon referred to as the "whip effect" is also pointed out. Some attention has been given to another aspect of the problem in section 5; the effect of the foundation on the rocking motion of a rigid structure is taken into account. It is shown that in this case the same methods using a spectrum and effectiveness factors can still be applied by introducing an additional degree of freedom and a natural period corresponding to the rocking motion on the foundation.

INTRODUCTION

To a great extent the design of earthquake resistant structures is still an art based on observational facts and experience. Development of experimental

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NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May, 1942.

[1] Research Assoc. in Aeronautics, California Inst. of Technology, Pasadena, Calif.; Asst. Prof. of Mechanics, Physics Dept., Columbia Univ. (on leave of absence).

[2] "A Mechanical Analyzer for the Prediction of Earthquake Stresses," by M. A. Biot, *Bulletin*, Seismological Soc. of America, Vol. 31, No. 2, April, 1941, pp. 151–171.

and analytical approaches has been very slow partly because of the lack of accurate information on the accelerations of strong-motion earthquakes and partly because of the great complexity of the phenomena involved. This gap between the empirical and scientific approaches is being reduced constantly, leading to improvement in codes and rules for the design of quake-resistant structures. Information on strong-motion earthquakes has been made available in recent years by the valuable work of the U. S. Coast and Geodetic Survey. Pooling the data furnished by the earthquake accelerograms with existing seismological knowledge only marks the beginning of analytical and experimental investigation. To picture the phenomenon in all its complexity one must imagine that the structure to be analyzed is floating on a medium in which highly irregular waves are propagating. The structure has the attributes of distributed elasticity and damping effects. Like a ship in the ocean it does not participate completely in the motion of the surrounding medium, its motion being dependent upon its own rigidity and mass and on its size relative to the waves. Internal friction and yield point in the surrounding soil must have an important effect on resonance phenomena. Also the properties of the surface layers of the earth vary greatly with location and depth so that complicated reflection, refraction, and diffraction of the waves must be expected. This is also true for agglomeration of buildings in which case the structures themselves must have a considerable influence on the intensity and direction of the waves.

Instead of approaching this problem as an entirely complex matter it must rather be expected that a solution will emerge gradually from the careful analysis of simplified cases in which the influence of each individual factor is clearly defined and checked critically against observation.

One of the simplifications usually introduced is the assumption that the ground behaves as a shaking table, the horizontal motion of which is taken to be the same as that derived from the horizontal accelerogram of an earthquake. A basic analytical approach to this problem was developed by the author in 1932,[3, 4, 5] in which the concept of the earthquake spectrum was introduced. This is a curve characteristic of a given earthquake which gives some kind of periodicity content by associating a certain acceleration intensity with a given period. It was shown in the earlier work how this curve could be computed and how use of it could be made for the acceleration of earthquake stresses. To this purpose, the motion of the structure is considered as the superposition of its various modes of vibration and the maximum stress produced in each mode is made to depend on a coefficient characteristic of the structure and of that particular mode. This coefficient in the present paper is referred to as the effectiveness factor. The procedure permits the easy comparison between various earthquakes and between various types of structures or modes within these structures as to the stresses produced by a given earthquake.

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[3] "Transient Oscillations in Elastic Systems," by M. A. Biot, *Thesis No. 259*. Submitted to the Aeronautics Dept., California Inst. of Technology, Pasadena, Calif., in 1932 in partial fulfilment of the requirements for the Degree of Doctor of Philosophy.

[4] "Theory of Elastic Systems Vibrating Under Transient Impulse with an Application to Earthquake-Proof Buildings," by M. A. Biot, *Proceedings*, National Academy of Science, Vol. 19 (1933), pp. 262–268.

[5] "Theory of Vibration of Buildings During Earthquakes," by M. A. Biot, *Zeitschrift für angewandte Mathematik und Mechanik*, Bd. 14, H. 4, (1934), pp. 213–223.

Applications of the concept of earthquake spectrum,[6] using the writer's analytical method, have been made by R. R. Martel, M. Am. Soc. C. E., and M. P. White, Assoc. M. Am. Soc. C. E., and experiments along the same line have been made by L. S. Jacobsen and N. J. Hoff.[7]

Notation.—The letter symbols in this paper are defined where they are first introduced and are assembled for reference in the Appendix.

I.—EFFECT OF AN EARTHQUAKE ON A STRUCTURE WITH ONE DEGREE OF FREEDOM

Consider a mass m connected to the ground by weightless springs (Fig. 1). The horizontal displacement of the mass relative to the ground is denoted by u, and the spring rigidity is such that a horizontal force F produces a displacement

$$u = \frac{F}{k} \qquad (1)$$

The constant k is called the "spring constant."

If the ground is given a horizontal acceleration a_0 applied very gradually so that no transient oscillation occurs, the mass will assume a constant deflection:

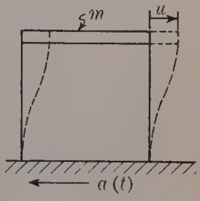

FIG. 1

$$u_0 = \frac{m\,a_0}{k} \qquad (2)$$

The total shear in the springs is then

$$V = m\,a_0 \qquad (3)$$

During an earthquake the horizontal acceleration is a function $a(t)$ of the time t. Denoting by v the displacement of the ground and neglecting the damping, the equation of motion of the mass m is

$$m\frac{d^2}{dt^2}(u + v) + k\,u = 0 \qquad (4)$$

The displacements u and v are taken positive to the right and the acceleration is taken positive to the left; hence

$$\frac{d^2v}{dt^2} = -a(t) \qquad (5)$$

and Eq. 4 may be written

$$m\frac{d^2u}{dt^2} + k\,u = m\,a(t) \qquad (6)$$

Eq. 6 shows that the relative displacement u obeys the differential equation of motion of a simple oscillator under the force

$$m\,a(t) = F(t) \qquad (7)$$

The earthquake is taken to start at the instant $t = 0$; the mass m being initially

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[6] "Some Studies on Earthquakes and Their Effects on Constructions," by R. R. Martel and M. P. White, Rept. on Earthquake Studies for Los Angeles County, Pt. I (1939) (unpublished).

[7] *Ibid.*, Pt. II.

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at rest, the relative displacement u as a function of time is given by the well-known solution[8]

$$u = \frac{1}{\sqrt{k\,m}} \int_0^t F(\theta) \sin \sqrt{\frac{k}{m}}\,(t - \theta)\, d\theta \qquad (8a)$$

or

$$u = \frac{T}{2\pi} \int_0^t a(\theta) \sin \frac{2\pi}{T}\,(t - \theta)\, d\theta \qquad (8b)$$

in which: $F(\theta)$ = force; θ = time variable of integration; and, T, the natural period of the oscillator, equals

$$T = 2\pi \sqrt{\frac{m}{k}} \qquad (9)$$

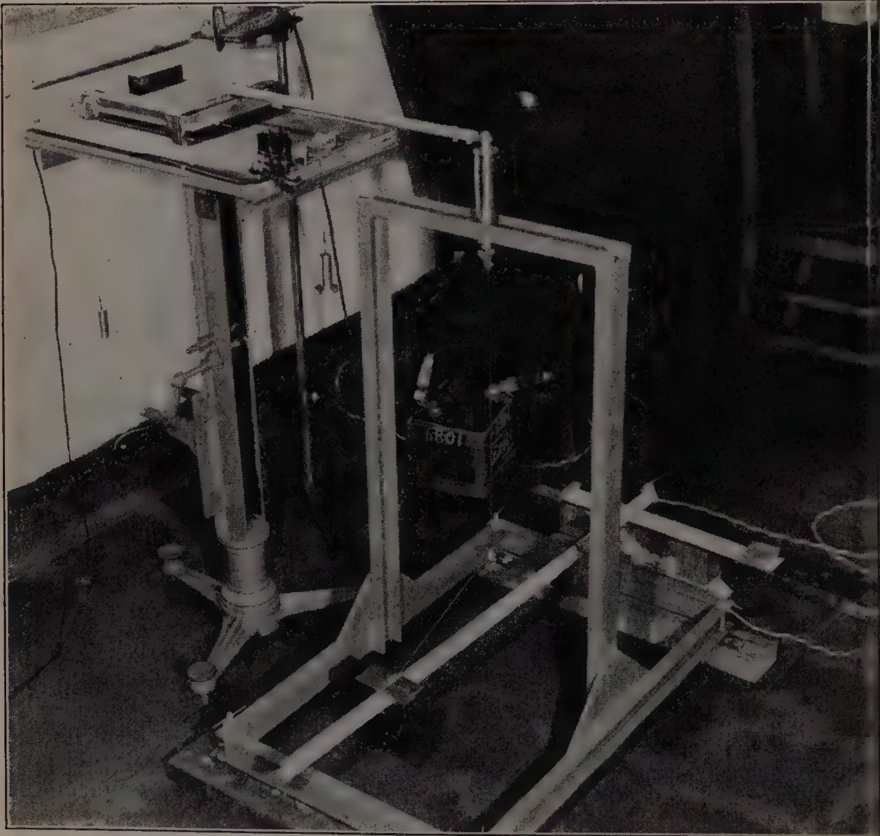

FIG. 2.—VIEW OF MECHANICAL ANALYZER

The total shear in the springs is

$$V = k\,u = m \frac{2\pi}{T} \int_0^t a(\theta) \sin \frac{2\pi}{T}\,(t - \theta)\, d\theta \qquad (10)$$

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[8] "Mathematical Methods in Engineering," by Theodor von Kármán and M. A. Biot, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, p. 404.

The quantity:

$$f(t) = \frac{2\pi}{T}\int_0^t a(\theta)\sin\frac{2\pi}{T}(t-\theta)\,d\theta \qquad (11)$$

is a function of time which gives the complete stress history of the oscillator. That is, if the integration is performed with respect to θ between the limits 0 and t, and repeated for all values of t, a function of time is obtained which, according to Eq. 10, will give the value of the total shear V at every instant t.

II.—Earthquake Spectrum Evaluated by Means of a Mechanical Analyzer

It is of special interest to find the maximum value of the stress produced by a given earthquake. Denoting by A the maximum value of Eq. 11, the maximum shear is written

$$V_{\max} = m\,A \qquad (12)$$

Comparing with Eq. 3, one may say that, as far as the maximum shear is concerned, the effect of the earthquake is equivalent to that of a constant acceleration A applied gradually at so slow a rate that only a statical deflection is produced without the occurrence of any transient oscillation.

Of course for a given earthquake the value of A depends on the parameter appearing in Eq. 11—that is, on the natural period T of the structure. The quantity A is referred to as the "effective acceleration" of the earthquake for the period T. It will be noticed that the effective acceleration for a particular earthquake depends only on the period of the oscillator. Therefore, one may evaluate this effective acceleration for various oscillator periods and consider it to be a characteristic function $A(T)$ of the period for each particular earthquake. This function is called the "acceleration spectrum" of the earthquake. (Use has sometimes been made of the expression "equivalent acceleration" for "effective acceleration," and "oscillator response curve" or "influence line for horizontal shear" instead of "spectrum.")

The engineering significance of this concept lies in the fact that, once the spectrum is known, it is possible to write immediately the value of the maximum shear produced by the earthquake on any undamped, one-degree-of-freedom, structure. To obtain the shear produced by an earthquake in such a structure of period T the mass of the structure is multiplied by the ordinate of the spectrum for the particular value T of the abscissa. Furthermore, as will be shown, it is possible to extend the usefulness of the spectrum to structures much more complicated than the one-degree-of-freedom oscillator considered herein.

It is relatively tedious to evaluate the spectrum by analytical methods, as this would involve the calculation of the integral (Eq. 11) from a graphically given accelerogram $a(t)$ and for a great number of values of both T and t. Fortunately there are simple experimental methods by which this can be done.

A mechanical analyzer shown in Fig. 2 has been developed for this purpose. A detailed description of the apparatus was given in a previous publication.[2] It is a torsion pendulum with variable tuning whose point of suspension can be made to turn proportionally to the acceleration of the earthquake. When the pendulum is tuned for the period T, it can be shown that its maximum amplitude yields the value of A. By varying the tuning it is then possible to plot the spectrum $A(T)$ point by point. The main advantages of this type of analyzer are its low cost, its simplicity of operation, and the fact that it takes an average of eight hours to plot one spectrum curve. The use of a torsion pendulum at the Bureau of Reclamation to evaluate earthquake stresses was mentioned by J. L. Savage, Hon. M. Am. Soc. C. E., in 1939.[9]

The following earthquakes have been analyzed (their spectrum curves are plotted in fractions of gravity g against period in seconds):

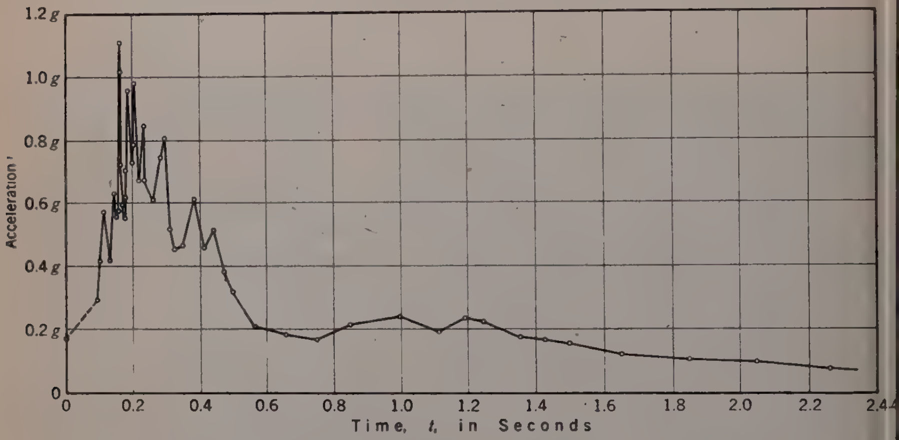

FIG. 3.—SPECTRUM OF EAST-WEST HORIZONTAL ACCELERATION OF THE EARTHQUAKE OF HELENA, MONT., OCTOBER 31, 1935

(*A*) Helena, Mont. (October 31, 1935) a horizontal east-west acceleration. The spectrum in Fig. 3 shows that a peak value of 1.05 g occurs for $T = 0.16$ sec. The maximum recorded acceleration of the earthquake is 0.16 g; and the amplification due to resonance is $\frac{1.05}{0.16} = 6.5$ times this value.

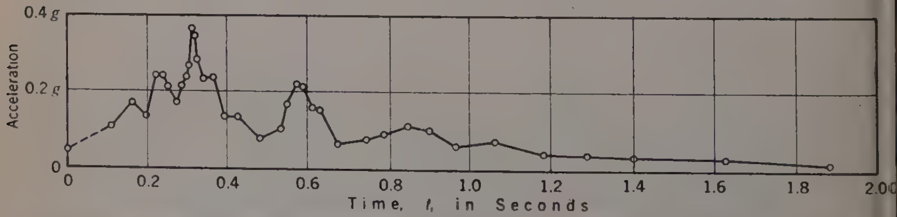

FIG. 4.—SPECTRUM OF THE NORTH-EAST HORIZONTAL ACCELERATION OF THE EARTHQUAKE OF FERNDALE, CALIF., FEBRUARY 6, 1937

(*B*) Ferndale, Calif. (February 6, 1937) a horizontal northeast acceleration. The spectrum in Fig. 4 shows that this is a minor earthquake. Its maximum intensity is 0.039 g, and the amplification is 9.5.

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[9] "Earthquake Studies for Pit River Bridge," by J. L. Savage, *Civil Engineering*, August, 1939, pp. 470–472.

(*C*) Ferndale (September 11, 1938) horizontal accelerations in both northeast and southeast directions. The maximum recorded intensity is 0.17 g in the northeast direction and the amplification is 6. Both spectrums are found to be analogous to (*A*).

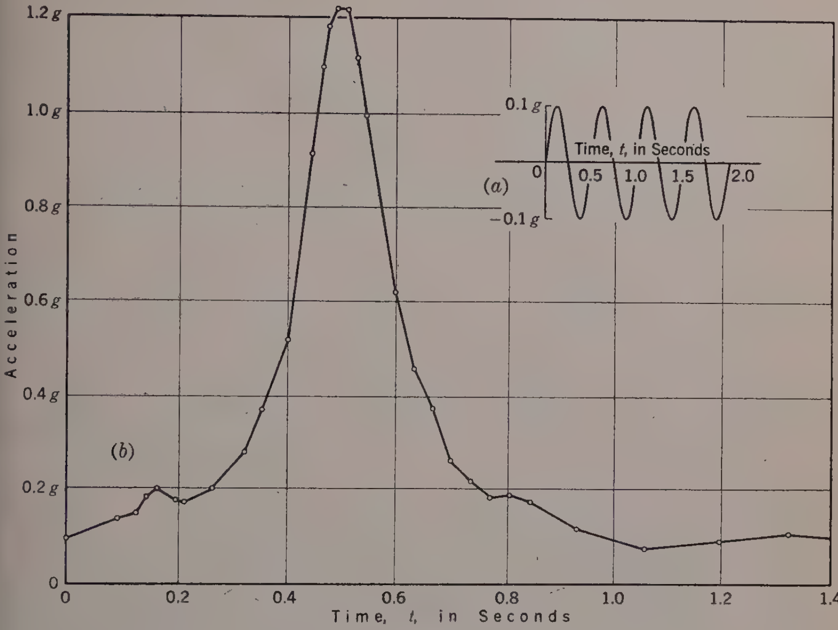

FIG. 5.—SPECTRUM OF AN ARTIFICIAL EARTHQUAKE CHARACTERIZED BY A SINUSOIDAL ACCELERATION

(*D*) A sinusoidal earthquake (Fig. 5(*a*)) of intensity 0.1 g and a duration of four cycles of 0.5 sec each was analyzed. Its spectrum is plotted in Fig. 5(*b*). The peak value of 1.23 g obtained is in good agreement with the theoretical value 1.25 g.

III.—SPECTRUM RELATED TO DESIGN AND ACTUAL BEHAVIOR OF STRUCTURES

It is apparent that, from the viewpoint of the designer, the individual sharp peaks in the spectrums are unimportant since their frequencies do not seem to be characteristic constants of the location as found by a comparison of the two Ferndale earthquakes (*B*) and (*C*) (see section II). The envelope of the spectrum, or better still, the envelope of a collection of spectrum curves obtained at the same location, constitutes the basic information for design purposes. A simple spectrum such as that plotted in Fig. 6 might well be taken to represent the Helena and Ferndale earthquakes ((*A*) and (*C*), section II). For $T > 0.2$ sec, the curve is chosen as the hyperbola

$$A = \frac{0.2\,g}{T} \quad\ldots\ldots\ldots\ldots\ldots\ldots(13)$$

which emphasizes in quantitative form the fact that buildings of longer periods undergo smaller stresses. In general, therefore, high buildings will be less vulnerable to earthquakes than the smaller structures with shorter periods. The accelerograms from which these spectrum curves have been derived were recorded with the instrument designed by the U. S. Coast and Geodetic Survey. Its natural period is 0.1 sec. Although it is properly damped to function as an accelerograph, too much significance must not be attached to that part of the spectrum for periods smaller than 0.2 sec.

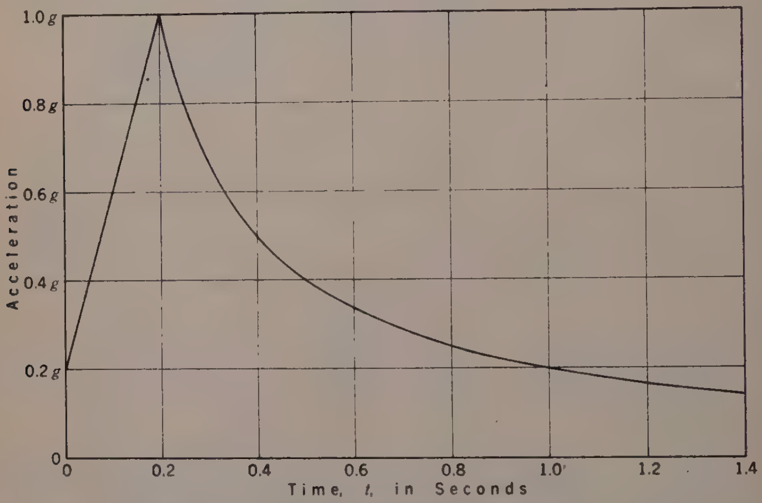

FIG. 6.—STANDARD SPECTRUM PROPOSED TO REPRESENT EARTHQUAKES (A) AND (C) FROM THE STANDPOINT OF DESIGN

Before proceeding any further it is important to give more careful consideration to the possible engineering interpretation of the foregoing results. The spectrums for earthquakes (*A*) and (*C*) indicate that an undamped structure with a period of approximately 0.2 sec would undergo a horizontal shear equal to its own weight. From observation of the effects of earthquakes this value seems rather high, but it must be remembered that it constitutes an upper limit and that actually a number of stress-reducing factors enter into play.

One of these factors, of course, is the damping influence of the structure. The magnitude of this damping effect must depend very much on the nature of the structure and the amplitude of the stresses. In fact, the damping observed with the aid of building vibrators or during minor earthquakes may very well be small, but one must be prepared to discover that in a strong earthquake the damping is considerably greater. When the amplitude of the stress reaches the yield point in some part of a structure, plastic deformation and friction will produce a high degree of energy dissipation. Assuming, for instance, that this type of damping occurs as soon as the spectrum ordinate is greater than 0.2 *g*, any further increase of stress by resonance will be strongly counteracted. That structural damping is an important factor agrees with the conclusion of studies by Professor White.[10]

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[10] "Friction in Buildings: Its Magnitude and Its Importance in Limiting Earthquake Stresses," by M. P .White, *Bulletin*, Seismological Soc. of America, Vol. 31, No. 2, April, 1941, pp. 93–99.

Another stress reduction is that due to the influence of the foundation. This influence is threefold. When oscillations are set up in a building, strains are produced in the foundation and energy is dissipated, by internal friction in the soil. This effect depends on looseness, shear strength, internal damping, etc. A second cause of stress reduction is the radiation of elastic waves into the soil because of the motion of the building. This phenomenon was the object of a theoretical investigation by K. Sezawa and K. Kanai.[11] By this effect the energy of the oscillations is drained from the building through the foundation and radiated into the soil in the form of elastic waves. The magnitude of this effect depends on the size and natural period of the structure and on the elastic constants and density of the surrounding soil. The elasticity of the foundation will have an influence on the stresses because it increases the natural period of vibration of buildings. In other words, the building will not follow the horizontal motion of the ground but, due to the elasticity of the foundation, it will tend to rock about its center of percussion. This effect is examined in more detail in section V, and is shown to be considerable.

Finally, it must be noted that a variation of the period with amplitude is very effective against resonance effect. The stress limitation due to this factor can be important especially in strong earthquakes when large deformations and local failures occur.

From the choppy aspect of the spectrum it may be concluded that slight differences in building periods may cause great differences in earthquake stresses. This may very well be one of the reasons for the paradoxical observation that the destructiveness of an earthquake varies greatly from one building to another at the same location. The differences are not as great as the spectrums would indicate, however, but this could be explained by taking into account the influence of the damping.

IV.—Effect of an Earthquake on a Structure with Many Degrees of Freedom

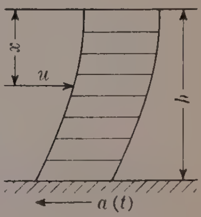

Fig. 7

The fact has been established that a number of modes of oscillation are excited by the earthquake, each of which contributes to the stress.[4, 5] It can be shown that each mode behaves like a system with a single degree of freedom and that the concept of spectrum is applicable to each mode separately. This will be demonstrated briefly by a simple example. Consider a building of height h, total mass m, and total rigidity k, both m and k being uniformly distributed (Fig. 7). The building is assumed to behave like a shear beam such that a unit horizontal displacement at the roof is produced by a force k. The shearing oscillations of the building obey the equation

$$h\,k\,\frac{\partial^2 u}{\partial x^2} + \frac{m}{h}\,a(t) = \frac{m}{h}\,\frac{\partial^2 u}{\partial t^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which u is the displacement relative to the ground at a distance x from the roof, and $a(t)$ is the horizontal acceleration of the ground, taken positive in a direction opposite to u. Write the acceleration $a(t)$ as a Fourier series in x

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[11] "Some New Problems of Forced Vibrations of a Structure," by K. Sezawa and K. Kanai, *Bulletin*, Earthquake Research Inst., Tokyo, Vol. XII, Pt. 4, December, 1934, p. 845; also "Decay in the Seismic Vibrations of a Simple or Tall Structure by Dissipation of Their Energy into the Ground," by K. Sezawa and K. Kanai, *ibid.*, Vol. XIII, Pt. 3, September, 1935, p. 681.

$$a(t) = \frac{4}{\pi}\left(\cos\frac{\pi x}{2h} - \frac{1}{3}\cos\frac{3\pi x}{2h} + \frac{1}{5}\cos\frac{5\pi x}{2h}\cdots\right)a(t)\ldots\ldots(15a)$$

and the solution $u(x,t)$ of Eq. 14 in the same form:

$$u(x,t) = U_1(t)\cos\frac{\pi x}{2h} + U_2(t)\cos\frac{3\pi x}{2h} + U_3(t)\cos\frac{5\pi x}{h}\ldots\ldots(15b)$$

The quantities U_1, $U_2\cdots$, represent the unknown roof deflections of each mode. Substituting Eqs. 15 into the differential Eq. 14 and equating the coefficients of $\cos\frac{\pi x}{2h}$, $\cos\frac{3\pi x}{2h}$, etc., to zero, the following equations are found:

$$\left.\begin{aligned} m\frac{d^2U_1}{dt^2} + \frac{k\pi^2}{4}U_1 &= \frac{4}{\pi}m\,a(t) \\ m\frac{d^2U_2}{dt^2} + \frac{9k\pi^2}{4}U_2 &= -\frac{4}{3\pi}m\,a(t) \\ \cdots\cdots\cdots&\cdots\cdots\cdots \\ m\frac{d^2U_n}{dt^2} + \frac{(2n-1)^2k\pi^2}{4}U_n &= \pm\frac{4m}{(2n-1)\pi}a(t) \end{aligned}\right\}\ldots\ldots(16)$$

These equations are analogous to Eq. 6 for the structure with a single degree of freedom. According to Eq. 8*a* the roof deflection of the *n*th mode is:

$$U_n(t) = \frac{16m}{(2n-1)^3\pi^3k}\,\frac{2\pi}{T_n}\int_0^t a(\theta)\sin\frac{2\pi}{T_n}(t-\theta)\,d\theta\ldots\ldots(17a)$$

in which T_n is the period of the *n*th mode. The largest value attained during the earthquake for the roof deflection in the *n*th mode can be written:

$$U_n = \frac{16m}{(2n-1)^3\pi^3k}A(T_n)\ldots\ldots(17b)$$

in which $A(T_n)$ is the ordinate at $T = T_n$ of the aforementioned acceleration spectrum. The shear stress

$$V = hk\frac{\partial u}{\partial x}\ldots\ldots(18a)$$

is maximum at the base, $h = x$; hence the maximum shear in the *n*th mode is

$$V_n = \frac{8}{(2n-1)^2\pi^2}m\,A(T_n)\ldots\ldots(18b)$$

Denoting by V_g the stress which would be produced by a statical horizontal acceleration equal to gravity

$$V_n = C_n\frac{A(T_n)}{g}V_g\ldots\ldots(18c)$$

in which

$$C_n = \frac{8}{(2\,n - 1)^2\,\pi^2} \quad \text{(19)}$$

The expression $\frac{A}{g}\,V_g$ represents the shear that would occur in a structure of one degree of freedom with the same mass and period as the particular mode considered. The factor C_n may be called an "effectiveness factor" because it represents the extent to which this shear appears in each mode of the structure with distributed mass and rigidity. A more direct method of calculating these factors is available. Assume that some restraining mechanism is used so that the building can only deflect in its fundamental mode, and apply a horizontal statical acceleration g. The deflection will be found by the energy principle. The external work, W, done by the inertia forces applied gradually is:

$$W = \frac{1}{2}\,\frac{m\,g\,U_1}{h}\int_0^h \cos\frac{\pi\,x}{2\,h}\,dx \quad \text{(20)}$$

The elastic potential energy E_p in this deformation is more conveniently evaluated as the kinetic energy in the fundamental mode of vibration; hence:

$$E_p = \frac{1}{2}\,\frac{m}{h}\left(\frac{2\,\pi}{T_1}\right)^2 U^2{}_1 \int_0^h \cos^2\frac{\pi\,x}{2\,h}\,dx \quad \text{(21)}$$

Equating expressions 20 and 21 gives

$$U_1 = \frac{16}{\pi^3}\,\frac{m\,g}{k} \quad \text{(22a)}$$

and the shear

$$V_1 = \frac{8}{\pi^2}\,m\,g = \frac{8}{\pi^2}\,V_g \quad \text{(22b)}$$

The factor $C_1 = \frac{8}{\pi^2}$ appears in this expression, and coincides, for $n = 1$, with the value (19) already found by the more elaborate previous method. The same method is applicable to the higher modes.

The values of the coefficients for the three first excited modes are: $C_1 = 0.810$; $C_2 = 0.0900$; and $C_3 = 0.0324$. Similarly coefficients can be found for all kinds of structures provided the natural modes of vibration are known. In the earlier work[4, 5] the effectiveness factors were computed for a building with an elastic first story. Some of these values are given in Table 1 for different values of the elasticity ratio $\frac{k_1}{k}$. The rigidity k_1 of the first story is

TABLE 1.—CONSTANTS C FOR VARIOUS ELASTICITY RATIOS

$\frac{k_1}{k}$	C_1	C_2	C_3
0	1	0	0
0.556	0.993	0.0295	0.00436
2.50	0.947	0.0712	0.0155
∞	0.810	0.0900	0.0324

the force necessary to produce a unit deflection of the second floor with respect to the ground, and k is the rigidity between the second floor and the roof. Note that for a rigid first story $\left(\frac{k_1}{k} = \infty\right)$ the coefficients are the same as the foregoing for a uniform building. These coefficients make possible a comparison of the relative importance of the various modes. In a building with uniform distribution of mass and rigidity, if 100% denotes the shear that would act in a structure with one degree of freedom under an earthquake of constant spectrum the fundamental mode of the uniform building picks up 81% of the shear, whereas the second and third modes pick up only 9% and 3% of the shear. Table 1 shows that the effect of an elastic story is to decrease, further, the importance of the higher modes relatively to the fundamental.

For the case of a cable or simple truss the maximum shear stress is also given by Eq. 18*c* with appropriate values of C_n. The bending moment in a truss is expressed as

$$M_n = B_n \frac{A}{g} (T_n)\, M_g \quad \text{.....................(23)}$$

in which M_g denotes the maximum bending moment produced by a static horizontal force equal to gravity. The coefficients C_n and B_n depend on the type of structure and the particular mode of vibration. Values of these coefficients for a cable and a pin-ended uniform truss are given in Table 2. Only the symmetrical modes (n odd) are excited by the earthquake so that the coefficients for modes of even order are zero. It is seen that the importance of the higher modes for the bending moment tends to decrease more rapidly than for the shear. Comparing the importance of the various modes under the assumption that the spectrum follows a law of the hyperbolic type as in Eq. 13, the conclusion may be drawn that generally the higher modes are less dangerous than the fundamental. Exception must be made for the case of the shear in a flexural beam where it seems that each mode would carry about the same amount of shear. This is due to the fact that the effectiveness coefficients for shear in a mode of oscillation of the order n decreases as $1/n^2$ whereas the frequency of a flexural beam increases approximately as n^2. However, in practice, due to the increasing influence of damping in the higher modes, one would expect that their importance would always be less than that of the fundamental. The attention of the reader is called to the fact that the values of the effectiveness factors for the higher modes of a truss[2, 12] are erroneous.

TABLE 2.—COEFFICIENTS C_n AND B_n

Order of mode	$n = 1$	$n = 2$	$n = 3$
C_n for cable....................	0.810	0.090	0.032
C_n for pin-ended uniform truss ..	0.810	0.090	0.032
B_n for pin-ended uniform truss ..	1.03	0.038	0.008

Table 2 was applied to the evaluation of an upper limit for the stresses that would be produced by an earthquake of the Helena or Ferndale type in the San Francisco-Oakland Bay Bridge.[12] Conditions are found to be least favorable in the side-span truss which is treated as a pin-ended truss with a period of 3 sec. For this period the effective acceleration of the standard spectrum in Fig. 6 is $A = 0.066\, g$. The effectiveness factor B_1 for the bending moment in the fundamental mode is $B_1 = 1.03$; hence the bending moment is

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[12] *Transactions*, Am. Soc. C. E., Vol. 106 (1941), p. 1385.

$$M_1 = 0.068\, M_g \quad (24)$$

This means that the maximum bending moment during the earthquake is not greater than that due to a static horizontal acceleration of 6.8% gravity. From the foregoing values of the effectiveness factors it seems as if the stresses in structures with more than one degree of freedom are of the same order as those in a structure with a single degree of freedom. This is not always true, however, and a particularly dangerous condition may arise from assuming that it is.

Consider, for instance, that a mass m_1 is elastically restrained to the ground with a stiffness k_1, and that a much smaller mass m_2 is restrained to m_1 with a stiffness k_2 (Fig. 8). Denoting by u_1 and u_2 the horizontal displacement of m_1 and m_2, respectively, the equations for the amplitudes of the harmonic motion are

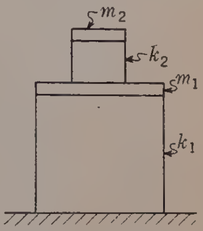

Fig. 8.—System with Two Degrees of Freedom, the Main Building and the Roof Structure

$$m_1 u_1 \omega^2 = k_1 u_1 + k_2 (u_1 - u_2) \quad (25a)$$

and

$$m_2 u_2 \omega^2 = k_2 (u_2 - u_1) \quad (25b)$$

in which ω is the circular frequency.

Assume that the small mass is "tuned" to the vibration of the large mass, by making

$$\frac{k_1 + k_2}{m_1} = \frac{k_2}{m_2} = \omega^2{}_1 \quad (26)$$

Writing $\frac{\omega}{\omega_1} = \lambda$ the equations of motion become

$$(\lambda^2 - 1)\, u_1 + \frac{m_2}{m_1} u_2 = 0 \quad (27a)$$

and

$$u_1 + (\lambda^2 - 1)\, u_2 = 0 \quad (27b)$$

Hence, by elimination of u_1 and u_2,

$$\lambda^2 - 1 = \pm \sqrt{\frac{m_2}{m_1}} \quad (28)$$

From Eqs. 27*b* and 28 the ratio of amplitudes is

$$\frac{u_2}{u_1} = \mp \sqrt{\frac{m_1}{m_2}} \quad (29)$$

The lower sign refers to the fundamental mode. Now, in order to evaluate the effectiveness factor a restraint is assumed to force the structure to deflect in its fundamental mode while a static horizontal acceleration equal to gravity is applied.

The work done by the gravity force in deflecting the structure is

$$\frac{g}{2}(m_1 u_1 + m_2 u_2) = \frac{1}{2} m_1 g u_1 \left(1 + \sqrt{\frac{m_2}{m_1}}\right) \quad \text{(30a)}$$

and the potential energy is

$$\frac{\omega^2}{2}(m_1 u^2_1 + m_2 u^2_2) = \frac{m_1}{m_2} k_2 u^2_1 \left(1 - \sqrt{\frac{m_2}{m_1}}\right) \quad \text{(30b)}$$

Equating these two expressions yields

$$u_1 = \frac{1}{2} \frac{m_2 g \left(1 + \sqrt{\frac{m_2}{m_1}}\right)}{k_2 \left(1 - \sqrt{\frac{m_2}{m_1}}\right)} \quad \text{(31a)}$$

and the shear in the spring k_2 is

$$V_1 = k_2 (u_2 - u_1) = \frac{1}{2}\left(1 + \sqrt{\frac{m_1}{m_2}}\right) m_2 g \quad \text{(31b)}$$

Therefore the effectiveness factor is

$$C_1 = \frac{1}{2}\left(1 + \sqrt{\frac{m_1}{m_2}}\right) \quad \text{(32)}$$

It is seen that for small values of the mass ratio $\frac{m_2}{m_1}$ this factor becomes quite large. For instance, if this mass ratio is $\frac{1}{100}$, then $C_1 = 5.5$. If an earthquake acts on such a system the stress in the spring k_2 will be

$$V_1 = 5.5 \frac{A}{g} (T_1) V_g \quad \text{(33)}$$

or 5.5 times what it would be if the spring mass (k_2 m_2) were directly connecte to the ground. Note that in Eq. 33, $V_g = m_2 g$ is the stress in the spring k under a horizontal statical acceleration equal to gravity. This introduces an example in which the effectiveness factor C_n refers to a particular location in the structure. It is then possible to refer to the effectiveness factor "at a certain point" in the structure. The amplification effect revealed here belong to a class of phenomena which might be designated as a "whip effect." Considered from the standpoint of wave propagation it is analogous to what hap pens in a whip, when a wave generated at the base is propagated to the tip. The wave may be considered as an energy lump moving from the heavy part of the whip to the lighter section at the tip. This produces at the tip a concentration of energy in a small mass and therefore gives rise to a high velocity. This effect is essentially associated with taper and explains many vibration failures in tapered beams, such as propeller tip failure or the fact that a tapered column receiving a blow at the base will fail at the tip. The case of two degrees of freedom, treated in the example, may well be considered as a simplified model of a tapered building. It points to the possibility of serious danger in roof structures and is in agreement with the observation of disproportionate damage in penthouses in the Long Beach earthquake of March, 1933. The California Building Code takes this danger into consideration by raising the lateral gravity force for which the penthouse must be designed.

The case of a structure in which the mass and stiffness both vary linearly with distance from the top was investigated by Professors Martel and White.[6] The effectiveness factor C_1 for total shear at various heights is:

Distance, $\frac{x}{h}$, from the roof	C_1
0	1.60
0.2	1.55
0.5	1.33
0.8	0.97
1.0 (base)	0.693

It is seen that these coefficients also point to the existence of a "whip" effect.

V.—Influence of Foundation on Motion of Blocks

The foregoing mathematical developments are based on the tacit assumption that the effect of an earthquake is the same as that of a shaking table. As will be shown now there is strong theoretical evidence that this concept must be modified to take care of the influence of the foundation. The elastic properties of the structure cannot be dissociated entirely from those of the ground and both must be studied simultaneously in order to predict the dynamic properties of the system.

The problem is extremely complex because it involves a complete knowledge of the propagation and properties of the seismic waves in the strongly heterogeneous surface layers of the earth as well as their diffraction and reflection by objects built on the surface. The influence of the soil on the damping of oscillations in structures was also discussed in the foregoing. An immediate answer to such a complex problem cannot be expected. It is believed, however, that the simplified problem noted in this section throws revealing light on the nature of foundation effects.

The question investigated is that of the influence of ground elasticity on the rocking motion of a block. How resistant is the surrounding soil to the rocking displacement of a foundation? What are the factors influencing this rigidity? Can this effect have a practical influence on the action of earthquakes on buildings? The problems are simplified by neglecting the mass of the soil the internal friction in the soil, and the radiation of elastic waves due to the rocking.

Assume that the coordinate axes xy lie on the surface of the soil. A load distribution $p(x)$-function only, of the distance x, is applied to the surface on a strip infinitely long in the y-direction and extending from $x = -L$ to $x = +L$. The distribution is asymmetric with respect to the y-axis so that

$$p(-x) = -p(x) \qquad (34)$$

It can be shown from the theory of elasticity that the soil deflection $w(x)$ is given by

$$w(x) = \frac{2}{\pi}\frac{(1-\mu^2)}{E}\int_0^L p(\xi)\log_e\left|\frac{x+\xi}{x-\xi}\right| d\xi \qquad (35)$$

in which E is Young's modulus of the soil and μ its Poisson ratio. Substitute in Eq. 35 the pressure distribution

$$p(x) = \frac{\alpha\, x}{\sqrt{L^2 - x^2}} \qquad (36)$$

Then, the deflection is a straight line:

$$w = 2\,\alpha\,\frac{(1-\mu^2)}{E}\,x \qquad (37)$$

In other words, the assumed pressure distribution (Eq. 36) is that under a rigid slab of width $2\,L$ rocking about the y-axis. The ratio between the elastic moment M of p about the axis and the slope $\frac{dw}{dx}$ is the elastic stiffness coefficient C_r for rocking motion

$$C_r = \frac{M}{\frac{dw}{dx}} = \frac{\pi}{4}\frac{E}{(1-\mu^2)}L^2 \qquad (38)$$

If a uniform pressure distribution equal to a constant p from $x = 0$ to $x = +L$ and $-p$ from $x = 0$ to $x = -L$ is applied, the deflection of the soil is

$$w = \frac{2}{\pi}\frac{(1-\mu^2)}{E}\,p\,L\left(\log_e\left|\frac{L+x}{L-x}\right| + \frac{x}{L}\log_e\left|\frac{L^2-x^2}{x^2}\right|\right) \qquad (39)$$

From this solution it is possible to derive, by superposition, the deflection due to uniform asymmetric load distribution extending from $x = \pm L$ to various distances from the axis. It is found that the average rocking stiffness is given by a formula similar to Eq. 38 except that the numerical coefficient is somewhat different from $\frac{\pi}{4}$. For instance, in the case in which the load extends from

$= 1$ to $x = \frac{4}{5} L$ and $x = -1$ to $x = -\frac{4}{5} L$ the coefficient $\frac{\pi}{4}$ is replaced by .231 π. It may be concluded that Eq. 38 represents a reasonable approximation, independent of the load distribution.

Eq. 38 implies the knowledge of the elastic constants E and μ of the soil. These are obtainable from tests made by M. Ishimoto and K. Iida.[13] Samples rom the underground of several regions within Tokyo, Japan, at depths to 0 ft, were made into the form of rectangular prisms and submitted to vibration ests. The average values of the Poisson ratio are found to be about $\mu = \frac{1}{3}$.

Some of the values found for Young's modulus are

Material	E, in lb per sq in.
Silty clay in natural state with 50% moisture	792
Same with 42% moisture	5,750
Loam	8,200

From measured velocities of waves in soils[14, 15] the following order of magnitudes may be derived:

Material	E, in lb per sq in.
Loose sand	4,000
Clay	6,000
Sandstone	100,000

D. D. Barkan[16] has experimented, under field conditions, with various foundations weighing as much as 30 tons and having various areas at the bottom as great as 90 sq ft. The foundation was tested dynamically for rocking oscillations and also statically for the rocking rigidity of the foundation slab on the soil. The results are in good agreement with Eq. 38. A length of 21 was taken to represent the size of the square foundation slab used in the test. The values of E derived from this test are

Material	E, in lb per sq in.
Loess	12,000
Water-soaked brown loam	5,000

The order of magnitude of these values is in satisfactory agreement with the aforementioned values cited from other sources.

It is now possible to derive the rocking period of oscillation of a block lying on the soil. The results established in the preceding sections have shown that the fundamental period of oscillation of a structure is one of the essenti controlling factors in earthquake stresses. The rocking period of a block is

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[13] "Determination of Elastic Constants of Soils by Means of Vibration Methods," by M. Ishimoto and K. Iida, *The Bulletin of the Earthquake Research Institute*, Vol. XIV (1936), Pt. I; also *ibid.*, Vol. XV (1937), Pt. II.

[14] "Soil Mechanics," by D. P. Krynine, McGraw-Hill Book Co., Inc., New York, N. Y., 1941.

[15] "Praktische Anwendungen der Baugrund Untersuchungen," by W. Loos, Berlin, J. Springer, 1935, p. 53.

[16] "Field Investigations of the Theory of Vibrations of Massive Foundations Under Machines," by D. D. Barkan, *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Vol. II, 1936, p. 285.

$$T = \frac{2\pi}{\sqrt{\frac{C_r}{m\,r^2} - \frac{g\,h}{r^2}}} \qquad \text{(40}$$

in which C_r is the stiffness of the foundation according to Eq. 38, m the mass the block, r the radius of gyration with respect to the rocking axis, and h th height of the center of gravity above the ground. The term $\frac{g\,h}{r^2}$ represents th destabilizing influence of gravity. The case in which

$$\frac{C_r}{m\,r^2} = \frac{g\,h}{r^2} \qquad \text{(41}$$

corresponds to statical instability when the tipping moment due to gravity equal to the restoring moment of the soil. Usually the term $\frac{g\,h}{r^2}$ may be neglected and if the weight $m\,g$ is written as $m\,g = 2\,p_s\,L$ the period becomes

$$T = 9.5\sqrt{\frac{p_s}{E}\,\frac{r^2}{g\,L}} \qquad \text{(42}$$

The pressure p_s is an average load on the foundation due to the weight of th structure. Consider the case of hard clay, and use for p_s the bearing capacit given by the Boston Buildin Code—p_s = 84 lb per sq in According to the aforecite data, a reasonable value for har clay seems to be E = 15,000 l per sq in. The periods given i Table 3 are then obtained. Wit these values of the periods th spectrum may be used to eva uate earthquake stresses as explained previously herein.

TABLE 3.—VALUES OF THE PERIOD T, IN SECONDS

2 L (ft)	VALUES OF r, IN FEET			
	5	10	50	100
5	0.39	0.78	3.9	7.8
10	0.28	0.55	2.8	5.5
50	0.12	0.24	1.2	2.4

According to the standard spectrum in Fig. 6 the stresses are inversel proportional to the period of the structure. Since the soil can have a marke effect on the period it is to be expected that it exerts also a proportional influence on the destructiveness of the earthquake. Actually, of course, structures do not behave as rigid blocks, so that their period will be influenced b both the building and the foundation rigidity. The combined periods must b used in applying the spectrum curves to evaluate earthquake stresses. According to the data in Table 3 and Eq. 42 the slenderness of a structure has considerable effect on its vulnerability to earthquakes. This probably bear some relation to the observed fact that columns and towers sometimes sho paradoxical resistance with respect to neighboring structures.

CONCLUSIONS

Experimental and analytical methods of approach have been developed for the evaluation of earthquake stresses. An earthquake may be characterized by a certain function of time, called a spectrum, which is derived from the accelerogram by means of a mechanical analyzer. By using the spectrum an upper limit can be evaluated simply for the stresses produced in undamped structures of given dynamic properties. The stresses in each mode are shown to be derived quite simply from the value of the period and a coefficient dependent on the nature of the structure and referred to as the "effectiveness factor."

The comparison of stresses calculated by this method with observed destructiveness of an earthquake on a building seems to indicate that, at least for short periods, the values of the stresses obtained are considerably higher than could be expected. The possible effect of certain stress-reducing factors is discussed, such as internal damping in the soil and in the structure, the radiation of elastic waves, etc. The influence of these factors can become very large in certain cases.

Particular attention is given to the influence of the elasticity of the foundation and it is shown that considerable stress reduction occurs through the lengthening of the natural period due to the foundation. This is equivalent to stating that rigid slender structures will have a tendency to rock about the center of percussion because of the elastic yielding of the foundation.

Attention is directed to a phenomenon referred to as the "whip" effect which increases the destructiveness of earthquakes on penthouses and the tip of tapered columns and buildings.

Spectrum curves presented herein indicate that the effectiveness of an earthquake is inversely proportional (roughly) to the period, so that increasing the period means an increase in safety. Application of the methods developed in this paper should be helpful as a guide in extending safety rules and interpreting earthquake data.

A considerable field lies open for further research, especially as regards the stress-reducing factors discussed in section III. The influence of damping, for instance, could be taken into account by using a controllable amount of damping in the mechanical analyzer. This would yield a set of "damped response curves" which would give directly the effective acceleration for a building with given period and damping. In this terminology the spectrum would correspond to the undamped response curve. Further research should also be directed toward a better knowledge of the dynamics of the foundation, the interference of structures with one another, and the influence of agglomerations on the earthquake waves themselves.

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APPENDIX

NOTATION

The following letter symbols, adopted for use in this paper, conform essen tially to Standard Letter Symbols for Mechanics, Structural Engineering, an Testing Materials, prepared by a Committee of the American Standar Association with Society representation and approved by the Association i 1932:[17]

A = effective acceleration giving the maximum value of shear throug Eq. 12: $A(T)$ = acceleration spectrum;
a = linear acceleration:
  a_0 = constant horizontal acceleration;
  $a(t)$ = earthquake acceleration as a function of time (accelero gram curve);
  $a(\theta)$ = acceleration as a function of the variable of integratio θ (Eq. 8b);
B_n = effectiveness factor for the bending moment in the nth mode (se Table 2 and Eq. 23); extent to which bending moment appears i each mode of the structure;
C_n = effectiveness factor for the shear in the nth mode (see Tables 1 an 2 and Eq. 18c); extent to which shear appears in each mode the structure;
C_r = coefficient defining the stiffness of a foundation (Eq. 38);
E = modulus of elasticity: E_p = potential energy;
F = force;
g = gravity constant;
h = height;
k = spring constant; rigidity: k_1 = the rigidity between ground and se ond floor;
L = length;
M = bending moment:
  M_n = maximum bending moment in the nth mode of a trus (Eq. 23);
  M_g = maximum bending moment produced by a static hor zontal force equal to gravity;
m = mass;
p = unit pressure; uniform pressure distribution:
  p_s = average load on a foundation due to the weight of th structure;
  $p(x)$ = pressure distribution under a rocking foundation at distance x from the axis;
  $p(\xi)$ = pressure distribution with the variable of integration in the place of x;

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[17] ASA–Z10a–1932.

r = radius of gyration;

T = period of vibration; natural period of oscillation:

  $A(T)$ = acceleration spectrum;

  T_n = the period of the nth mode;

t = time;

$U_n = U_n(t)$ = largest value of roof deflection obtained during an earthquake for the nth mode, as a function of time;

u = displacement relative to the ground (u and v are taken positive to the right):

  u_0 = a constant deflection (Eq. 2);

  $u(x,t)$ = displacement at the instant t and distance x from the roof;

V = total shear in a structure:

  V_n = maximum shear in the nth mode of a cable or truss;

  V_g = maximum shear produced by a static horizontal force equal to gravity;

v = (see u);

W = work;

$w(x)$ = soil deflection;

x = distance from roof (Eq. 14); distance from rocking axis of foundation (Eq. 35);

α = (Eq. 36) a coefficient;

θ = variable of integration in Eqs. 8;

λ = ratio of natural frequency ω to ω_1 (Eq. 27a);

μ = Poisson's ratio;

ξ = variable of integration in place of x (Eq. 35);

ω = circular frequency: ω_1 = circular frequency defined by Eq. 26.

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PAPERS

VARIATION OF ELASTIC CHARACTERISTICS ON STATICALLY INDETERMINATE QUANTITIES

By A. Hrennikoff,[1] Assoc. M. Am. Soc. C. E.

Synopsis

As is well known, the values of statically indeterminate quantities are influenced by the elastic characteristics of the structure, such as cross-sectional areas of compression and tension members, moments of inertia of flexural members, and moduli of elasticity. The actual magnitudes of these elastic characteristics are likely to differ somewhat from the values assumed in analysis because of inaccuracies and accidents of manufacturing and construction, and these may result in deviation of the actual stress values from the quantities used in the design. A proportional change in an elastic characteristic in different parts of the structure in most cases will not affect the values of static unknowns, but a random variation of cross-sectional area, moment of inertia, or modulus of elasticity from point to point may influence the results considerably, depending on location, magnitude, and sign of the variation.

These discrepancies or variations of the elastic characteristics are not always negligible. Thus, a $\frac{1}{4}$-in. deficiency in the thickness of a 12-in. reinforced-concrete slab causes a decrease of more than 6% in the moment of inertia of the member. The modulus of elasticity of steel is not likely to differ more than 3% from the usual value of 29,000,000 lb per sq in., but the modulus of concrete is subject to large variations. Thus, in tests of full-sized reinforced-concrete rigid frames, conducted in 1938,[2] the values of the modulus of elasticity of concrete specimens cut from different parts of the same frame varied from 3.7 to 5.9 million pounds per square inch—that is, the extreme values differed from the average of 4.8 by 23%. In conditions of actual practice these variations are likely to be even greater.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **May, 1942.**

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[2] "An Investigation of Rigid Frame Bridges," by Wilbur M. Wilson, Ralph W. Kluge, and John V. Coombe, *Bulletin No. 308*, Univ. of Illinois Eng. Experiment Station, 1938, Pt. 2.

This paper presents a general method of determining the greatest possible variation in the magnitude of a statically indeterminate quantity caused by most unfavorable distribution of assumed legitimate variations in the elastic characteristics.

NOTATION

The letter symbols used in this paper are defined where they first appear and are assembled for convenience of reference in the Appendix.

MAXIMUM VARIATION OF A STATICALLY INDETERMINATE QUANTITY

Before proceeding with the mathematics of the problem, it is necessary to state definitely what variations of the elastic characteristics should be considered legitimate. It will be assumed that an elastic characteristic may deviate from its theoretical value ϵ anywhere on the structure by an amount not in excess of $i\,\epsilon$, in which i is a small, theoretically infinitesimal, quantity. The actual values of the assumed variations and their signs at different points of the structure will be selected in such a manner as to make the variations of the statically indeterminate quantity in question the greatest. A different statically indeterminate quantity will require a new arrangement of variations of the elastic characteristic, staying within the limits $\pm\, i\,\epsilon$ anywhere on the structure. It will be assumed also that the modulus of elasticity E of flexural members will vary in such a manner as to have the same values in the planes normal to the axis of the member. Thus the member will consist of short rectangular blocks, cut by sections normal to the axis, with E varying from block to block but constant in the same block. This assumption that E is constant in each block is more severe in its effect on the variation of the statically indeterminate quantity than the assumption that it changes across the axis of the member, since a given change in E over the entire block modifies flexural deformability of this particular element of the structure by a greater amount than an equal change in E extending only over a part of the block.

Consider a structure with an arbitrary number of statically indeterminate quantities acted upon by a given set of loads. For definiteness only three static unknowns X_a, X_b, and X_c will be used in the derivation, although the formulas derived will be applicable to any number of unknowns. The equations of continuity from which the unknowns are to be found may be written in the form:

$$\left.\begin{aligned} X_a\,\delta_{aa} + X_b\,\delta_{ab} + X_c\,\delta_{ac} + \delta_{ao} &= 0 \\ X_a\,\delta_{ab} + X_b\,\delta_{bb} + X_c\,\delta_{bc} + \delta_{bo} &= 0 \\ X_a\,\delta_{ac} + X_b\,\delta_{bc} + X_c\,\delta_{cc} + \delta_{co} &= 0 \end{aligned}\right\} \dots\dots\dots\dots\dots (1)$$

In Eqs. 1 the familiar quantities δ refer to the basic statically determinate structure and represent the movements of the points of application of the static unknowns in the directions of the unknowns. The first subscript always indicates the point that moves, and the second subscript, if it is o, indicates that the movement is caused by the actual loads, and, if it is a, b, or c, signifies that the movement is caused respectively by any of the forces $X_a = 1$, $X_b = 1$, or $X_c = 1$. Thus, δ_{ab}, for example, represents the movement of the point of application of X_a in the direction of X_a, caused in the statically determinate structure by the force $X_b = 1$; also δ_{co} represents the movement of the point of application of X_c in the direction of X_c caused in the statically determinate structure by the actual loading. By the law of reciprocity $\delta_{ab} = \delta_{ba}$, with two similar equalities for the other values of δ.

By solving Eqs. 1,

$$X_a = -\frac{D_{1a}\,\delta_{ao} + D_{2a}\,\delta_{bo} + D_{3a}\,\delta_{co}}{D} \quad \text{(2)}$$

with two similar expressions for X_b and X_c.

In Eq. 2, D is the determinant composed of the coefficients of the three unknowns in Eqs. 1; and D_{1a}, D_{2a}, and D_{3a} are the corresponding minor determinants composed of the coefficients of the unknowns other than X_a—that is:

$$D = \begin{vmatrix} \delta_{aa} & \delta_{ab} & \delta_{ac} \\ \delta_{ab} & \delta_{bb} & \delta_{bc} \\ \delta_{ac} & \delta_{bc} & \delta_{cc} \end{vmatrix} \quad \text{(3)}$$

and

$$\left.\begin{aligned} D_{1a} &= \begin{vmatrix} \delta_{bb} & \delta_{bc} \\ \delta_{bc} & \delta_{cc} \end{vmatrix} \\ D_{2a} &= \begin{vmatrix} \delta_{bc} & \delta_{cc} \\ \delta_{ab} & \delta_{ac} \end{vmatrix} \\ D_{3a} &= \begin{vmatrix} \delta_{ab} & \delta_{ac} \\ \delta_{bb} & \delta_{bc} \end{vmatrix} \end{aligned}\right\} \quad \text{(4)}$$

The minor determinants D_{1b}, D_{2b}, etc., entering the expressions for X_b and X_c, are formed in a manner similar to Eq. 4.

It will be recalled that the various expressions δ represent integrals involving the elastic characteristics. Thus, in structures consisting of flexural members only,

$$\left.\begin{aligned} \delta_{ab} &= \int \frac{m_a\,m_b}{E\,I}\,ds \\ \delta_{ao} &= \int \frac{M_o\,m_a}{E\,I}\,ds \end{aligned}\right\} \text{(etc.)} \quad \text{(5a)}$$

In Eq. 5*a*, I is the moment of inertia; and M_o, m_a, and m_b signify the bending moments developed in the basic statically determinate structure: By the given loading; by the force $X_a = 1$; and by the force $X_b = 1$, respectively. Likewise, in structures made up of the direct-stress members only:

$$\left.\begin{aligned} \delta_{ao} &= \sum \frac{N_o\,n_a}{E\,A}\,L \\ \delta_{ab} &= \sum \frac{n_a\,n_b}{E\,A}\,L \end{aligned}\right\} \text{(etc.)} \quad \text{(5b)}$$

In Eq. 5*b*, L is the length of a structural member; and N and n are the direct stresses in the members corresponding to the same loading conditions as the moments M and m in Eq. 5*a*.

Suppose now that one of the elastic characteristics, such as the modulus of elasticity E, increases by an infinitesimal increment dE in a block of the length ds in any part of the structure. This causes differential increases in δ-values —$d\delta_{ao}$, $d\delta_{ab}$, etc. Since Eqs. 1 still hold for the structure, with the elastic properties modified in this manner, the values of static unknowns must readjust themselves to the new δ-values, and their corresponding increments dX_a, dX_b, etc., may be found by taking the total differentials of Eqs. 1 and solving the resultant equations for these increments. By this differentiation:

$$\delta_{aa}\,dX_a + \delta_{ab}\,dX_b + \delta_{ac}\,dX_c + (X_a\,d\delta_{aa} + X_b\,d\delta_{ab} + X_c\,d\delta_{ac} + d\delta_{ao}) = 0 \quad (6)$$

with two other similar equations.

These three equations can be solved for the unknowns dX_a, dX_b, and dX_c since all values of X and δ are known from Eqs. 2 and 5, and values of $d\delta$ are fixed by the assumed variation in the elastic characteristic.

Next, introduce the new functions:

$$\left.\begin{aligned}\Delta_a &= X_a\,\delta_{aa} + X_b\,\delta_{ab} + X_c\,\delta_{ac} + \delta_{ao}\\ \Delta_b &= X_a\,\delta_{ab} + X_b\,\delta_{bb} + X_c\,\delta_{bc} + \delta_{bo}\\ \Delta_c &= X_a\,\delta_{ac} + X_b\,\delta_{bc} + X_c\,\delta_{cc} + \delta_{co}\end{aligned}\right\} \quad (7)$$

in which the X-quantities are assumed constant and equal to the values determined from Eq. 2; and the δ-quantities, defined by Eqs. 5, are thought of as functions of the elastic characteristics E and I, considered as variables. The new expressions Δ will be recognized as the left-hand side parts of the basic Eqs. 1, and, although they vanish for the original δ-values, they are generally different from zero for the modified values of δ corresponding to new and variable values of E and I.

By differentiation of Eqs. 7

$$d\Delta_a = X_a\,d\delta_{aa} + X_b\,d\delta_{ab} + X_c\,d\delta_{ac} + d\delta_{ao} \quad (8)$$

with two similar expressions for $d\Delta_b$ and $d\Delta_c$. Substitution of these expressions in Eqs. 6 converts the latter into the following form:

$$\left.\begin{aligned}\delta_{aa}\,dX_a + \delta_{ab}\,dX_b + \delta_{ac}\,dX_c + d\Delta_a &= 0\\ \delta_{ab}\,dX_a + \delta_{bb}\,dX_b + \delta_{bc}\,dX_c + d\Delta_b &= 0\\ \delta_{ac}\,dX_a + \delta_{bc}\,dX_b + \delta_{cc}\,dX_c + d\Delta_c &= 0\end{aligned}\right\} \quad (9)$$

A marked similarity may be noted between these equations, serving for the determination of the variations dX_a, dX_b, and dX_c, and the basic Eqs. 1, serving for the determination of the static unknowns X_a, X_b, and X_c. The expressions for dX_a, etc., may be written by analogy with Eq. 2:

$$dX_a = -\frac{D_{1a}\,d\Delta_a + D_{2a}\,d\Delta_b + D_{3a}\,d\Delta_c}{D} \quad (10)$$

with similar expressions for dX_b and dX_c. The determinants D, D_{1a}, etc., have the same significance in these expressions as in Eq. 2.

Eq. 10 determines the value of a small variation in the static unknown X_a produced by an assumed variation in an elastic characteristic resulting in corresponding variations in the values of δ and Δ. When the modulus of elasticity E receives an increment dE anywhere along an infinitesimal length ds of one of the members comprising the structure, the changes caused in different values of δ may be exemplified by the following expression for one of them:

$$d\delta_{aa} = d\left(\frac{m^2_a}{E\,I}\,ds\right) = -\frac{m^2_a\,ds}{I}\frac{dE}{E^2} \quad \text{.................(11)}$$

If $\frac{dE}{E} = i'$, a small, theoretically infinitesimal, quantity, the following equation results:

$$d\delta_{aa} = -\,i'\,\frac{m^2_a\,ds}{E\,I} \quad \text{......................(12a)}$$

In Eq. 12*a* the variable quantities m_a, E, and I refer to the element of structure whose modulus of elasticity E has undergone the change.

If the variation of E occurs over a finite part, or even over the entire structure, the expression for $d\delta_{aa}$ becomes:

$$d\delta_{aa} = \int -\,i'\,\frac{m^2_a\,ds}{E\,I} \quad \text{.....................(12b)}$$

in which the integration is extended over the appropriate lengths of the members of the structure. Variations of other values of δ have similar expressions. The term i' should be considered at the present stage as a quantity variable along the structure but, naturally, the same in all values of δ for the same length element ds.

Substituting Eq. 12*b* in Eq. 8, and combining several integrals into one, the following expression results for $d\Delta_a$:

$$d\Delta_a = \int -\,i'\left(X_a\frac{m^2_a}{E\,I} + X_b\frac{m_a\,m_b}{E\,I} + X_c\frac{m_a\,m_c}{E\,I} + \frac{M_o\,m_a}{E\,I}\right)ds \quad \text{...(13)}$$

Two similar expressions follow for $d\Delta_b$ and $d\Delta_c$.

It should be stated that if the variation in E has occurred over the entire structure, and if i' is constant, the expressions for $d\Delta_a$, $d\Delta_b$, and $d\Delta_c$ all vanish since i' appears outside of the integral signs, and the integrals themselves become the left-hand parts of the basic Eqs. 1, slightly rearranged. On the other hand, with the value of i' variable, the expressions (Eq. 13) generally do not vanish.

Substituting the expressions for $d\Delta_a$, $d\Delta_b$, and $d\Delta_c$ in Eq. 10 and combining the integrals once more, the expression for dX_a becomes:

$$\begin{aligned} dX_a = \frac{1}{D}\int i'\Big[& D_{1a}\left(X_a\frac{m^2_a}{E\,I} + X_b\frac{m_a\,m_b}{E\,I} + X_c\frac{m_a\,m_c}{E\,I} + \frac{M_o\,m_a}{E\,I}\right) \\ & + D_{2a}\left(X_a\frac{m_a\,m_b}{E\,I} + X_b\frac{m^2_b}{E\,I} + X_c\frac{m_b\,m_c}{E\,I} + \frac{M_o\,m_b}{E\,I}\right) \\ & + D_{3a}\left(X_a\frac{m_a\,m_c}{E\,I} + X_b\frac{m_b\,m_c}{E\,I} + X_c\frac{m^2_c}{E\,I} + \frac{M_o\,m_c}{E\,I}\right)\Big]\,ds \quad \text{...(14)} \end{aligned}$$

The variable term i' in this expression has been moved outside of the brackets because it has the same value in all three quantities in parentheses, at the same elements ds.

Eq. 14 can be used to evaluate the variation dX_a provided that the function i' expressing the law of variation of dE along the structure is known or has been assumed. The problem, however, is to determine the greatest possible value of dX_a.

A little thought will serve to show that, if i' is taken as a constant quantity over the entire structure, it appears as a factor outside of the integral sign, and the integral itself then vanishes, since it represents a sum of three integrals, each of which vanishes separately. This deduction is evident, of course, from general principles, because a constant i' signifies a change in E in a constant ratio over the entire structure, and such change, as has been explained, does not generally influence the values of the static unknowns. This vanishing of the integral in Eq. 14 for a constant value of i' means, of course, that the values within the brackets must be positive for some parts of the structure and negative for others. Therefore, a comparatively large variation dX_a will correspond to an assumption of a positive i' in the parts of the structure in which the values within the brackets in Eq. 14 are positive, and a negative i' in the parts in which these brackets are negative. On this assumption, the integral in Eq. 14 will represent an infinite sum of infinitesimal terms each of which is positive. Furthermore, since each of these infinitesimal terms is proportional to i', the greatest possible variation dX_a will evidently correspond to an assumption of numerically greatest value of i' (namely i), occurring over the entire structure and being positive at the points with positive values within the brackets, and negative at the negative values within the brackets. Therefore,

$$dX_a\,(\max) = \frac{i}{D}\int\Bigg|\Bigg[D_{1a}\left(X_a\frac{m^2_a}{E\,I} + X_b\frac{m_a\,m_b}{E\,I} + X_c\frac{m_a\,m_c}{E\,I} + \frac{M_o\,m_a}{E\,I}\right)$$
$$+ D_{2a}\left(X_a\frac{m_a\,m_b}{E\,I} + X_b\frac{m^2_b}{E\,I} + X_c\frac{m_b\,m_c}{E\,I} + \frac{M_o\,m_b}{E\,I}\right)$$
$$+ D_{3a}\left(X_a\frac{m_a\,m_c}{E\,I} + X_b\frac{m_b\,m_c}{E\,I} + X_c\frac{m^2_c}{E\,I} + \frac{M_o\,m_c}{E\,I}\right)\Bigg]\Bigg|\,ds \quad \ldots\ldots(15)$$

in which the sign | | signifies the numerical value of the expression within the brackets.

Evaluation of Eq. 15 involves a determination of the zero points of the function within the brackets by equating it to zero and by solving the resultant equation in s, exactly or approximately. The integration then is performed separately in the intervals between the terminal points and these zero points, and the numerical values of the positive and negative parts of the integral are added. The function within the brackets may also be plotted against the abscissas s, and the value of the integral may be obtained by measuring the area between the curve and the axis of s with a planimeter. In this operation the areas both above and below the axis must be taken as positive.

It will be clear from examination of Eq. 15 that the function under the integral represents a first-degree polynomial with constant coefficients with respect to the terms $\frac{M_o\, m_a}{E\, I}$, $\frac{m^2_a}{E\, I}$, etc. Therefore, the integration in Eq. 15 is usually no more complicated than the integrations required for a determination of various δ-values. If the axis of the structure is of a complicated shape and if the loading is complicated, or if the moment of inertia is variable, an exact integration of Eq. 15 is impossible and that formula must be replaced by a summation, a procedure similar to the usual method of determining the values of δ in a statically indeterminate analysis.

After Eq. 15 has been evaluated it is useful to state the required variation in the form

$$dX_a\,(\max) = K\, i\, X_a \ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

and to calculate the coefficient K representing the maximum percentage by which the value of X_a may change if the elastic characteristic is to vary by 1%.

The method of determining the most unfavorable effect of a variation of the moment of inertia I is similar to the foregoing, except that the symbols i' and i would in this case signify the ratio $\frac{dI}{I}$ instead of $\frac{dE}{E}$.

In the analysis of statically indeterminate reinforced-concrete structures it is customary to make a constant percentage allowance for steel reinforcement in the moments of inertia and to use the moments of inertia of uncracked sections. Taking $I = 1.1\,\frac{b\, h^3}{12}$ and differentiating this expression

$$\frac{dI}{I} = \frac{db}{b} + 3\,\frac{dh}{h} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

It is reasonable to assume that the ratios $\frac{db}{b}$ and $\frac{dh}{h}$ can take any values up to the limits i_b and i_h anywhere on the structure, or in symbols

$$\left|\frac{db\,(\max)}{b}\right| = i_b; \qquad \text{and} \qquad \left|\frac{dh\,(\max)}{h}\right| = i_h \ldots\ldots\ldots (18a)$$

and

$$\left|\frac{dI\,(\max)}{I}\right| = i_b + 3\, i_h \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18b)$$

Eq. 18b must be substituted for i in Eq. 15 to evaluate the maximum effect of variation in I on the value of the statically unknown quantity X_a. In barrel arches the moment of inertia may vary only by virtue of a variation in the depth h, which requires that $i_b = 0$.

It is also possible to assume that

$$|db\,(\max)| = C_b \qquad \text{and} \qquad |dh\,(\max)| = C_h \ldots\ldots\ldots (19)$$

in which C_b and C_h are constant numbers. In that case the ratio i' in Eq. 14 becomes variable along the structure and remains under the integral sign.

It is needless to state that the method described herein is equally applicable to structures composed of tension and compression members. In that case integrals in Eqs. 12 to 15 must be replaced by the finite sums of the terms involving the functions $\frac{N_o\, n_a}{E\, A} L$, $\frac{n^2_a}{E\, A} L$, etc.

The greatest effect on X_a of the simultaneous most unfavorable variations in E and I for the same condition of loading may be found by direct addition of the two separate effects of changes in elastic characteristics. This can be demonstrated easily by reference to the foregoing equations. By the rules of differentiation the total differential of any δ when both E and I vary is equal to the sum of the two $d\delta$'s corresponding to separate variations of E and I. Then it follows from Eq. 8 that the quantity $d\Delta$, with any subscript for the condition when E and I both vary, equals the sum of the functions $d\Delta$ with the same subscript found for the conditions of separate variations in E and I. Finally, the linear form with respect to various $d\Delta$'s of Eq. 10 for the total variation dX_a proves the required theorem of straight addition of the two separate variations dX_a.

Eq. 15 for the greatest value of dX_a is perfectly general with regard to the number of static unknowns in the structure investigated, with the exception of the structure with only one unknown, since in that case symbols for minor determinants D_{1a}, etc., become meaningless.

For a structure only once statically indeterminate, $X_a = -\frac{\delta_{ao}}{\delta_{aa}}$, which by differentiation gives

$$dX_a = -\frac{1}{\delta_{aa}}\left(d\delta_{ao} - \frac{\delta_{ao}}{\delta_{aa}}\, d\delta_{aa}\right) \dots\dots\dots\dots\dots\dots (20a)$$

and then, following the same logic,

$$dX_a\ (\max) = \frac{i}{\delta_{aa}} \int \left| \left(\frac{M_o\, m_a}{E\, I} + X_a \frac{m^2_a}{E\, I} \right) \right| ds \dots\dots\dots (20b)$$

In a structure with two static unknowns, substitution of the values of determinants in Eq. 15 results in a simplified expression for the maximum variation:

$$dX_a\ (\max) = \frac{i}{\delta_{aa}\, \delta_{bb} - \delta^2_{ab}} \int \left| \delta_{bb} \left(\frac{M_o\, m_a}{E\, I} + X_a \frac{m^2_a}{E\, I} + X_b \frac{m_a\, m_b}{E\, I} \right) \right.$$
$$\left. - \delta_{ab} \left(\frac{M_o\, m_b}{E\, I} + X_a \frac{m_a\, m_b}{E\, I} + X_b \frac{m^2_b}{E\, I} \right) \right| ds \dots\dots\dots\dots (21)$$

MAXIMUM VARIATION OF STRESS QUANTITIES DEPENDENT ON STATIC UNKNOWNS

The greatest variations of stress quantities other than those chosen as static unknowns may be determined by two methods. In the first method different quantities are selected as static unknowns, including the stress quantities whose maximum variations are required, and then the problem is solved in the usual manner by the formulas presented herein.

In the second method, which is applicable only in cases when the variations of the original static unknowns have already been found for the same load condition, the variation of the new stress quantity is determined easily by differentiating the corresponding equation of statics. This may be best illustrated in an example.

Fig. 1 represents a fixed-ended rigid frame loaded on the horizontal part with a uniform load of intensity w. The structure is three times statically indeterminate. The bending moments at points A, B, and C, designated by X_a, X_b, and X_c, respectively, have been chosen as the unknowns, and their variations have been determined by Eq. 15. It is required to determine the greatest variation in M_E, the bending moment at the point E, the center of the horizontal part. By statics:

$$M_E = \frac{X_b + X_c}{2} + \frac{w\,L^2}{8} \qquad (22)$$

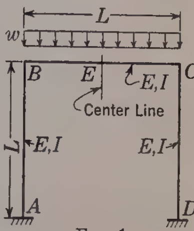


FIG. 1

Differentiating Eq. 22 and remembering that w and L are constant,

$$dM_E = \tfrac{1}{2}\,(dX_b + dX_c) \qquad (23a)$$

The three differentials in Eq. 23a may be considered as variations of the corresponding stress quantities. Since variations dX_b and dX_c are interdependent, it is incorrect to assign to them simultaneously the greatest values determined by Eq. 15. On the other hand, attributing to them the general integral expressions by Eq. 14 (in which the signs of the variations dE or dI in the variable terms i' at different points of the structure have not yet been specified) is quite legitimate. As a result of this substitution the following expression is obtained for the required greatest variation in M_E:

$$dM_E\,(\text{max}) = \frac{i}{D}\int \left|\tfrac{1}{2}\,[^b] + \tfrac{1}{2}\,[^c]\right| ds \qquad (23b)$$

in which the symbols $[^b]$ and $[^c]$ represent the brackets following the integral signs in the expressions for dX_b and dX_c.

MAXIMUM VARIATIONS FOR DIFFERENT LOAD CONDITIONS

Calculation of the maximum variation of a statically unknown quantity for a new load condition requires considerably less labor than a similar calculation for the first load, since the values of δ with subscripts a, b, and c, and the values of the determinants, are the same as in the first calculation. The quantities δ_{ao}, δ_{bo}, δ_{co} and the values of static unknowns X_a, X_b, and X_c are different, however.

The greatest variation dX_a, corresponding to two sets of loads acting simultaneously, may be found by superposing the results caused by separate actions of the two sets. However, it is not the numerical values of dX_a (max) that are to be superimposed but the expressions within the brackets in Eq. 15. The proof of this proposition may be derived by close inspection of the foregoing formulas. First, it should be realized that the expressions for the values of the moments M_o in the basic statically determinate structure, when both sets of loading are acting, may be obtained by superposition or algebraic addition of the values corresponding to separate sets. With this fact in mind it is easy to see that the quantities δ_{ao}, δ_{bo}, and δ_{co} (see Eq. 5a) also may be found by superposition, as concerns both the algebraic expressions under the integral signs and the numerical values obtained after the integration of these expressions. It is a well-known fact, evident also from Eq. 2, that the values of statically indeterminate quantities when two sets of loads are acting on the structure at the same time are equal to algebraic sums of the values of these quantities corresponding to separate actions of the loads. The last two statements, together with the fact that all δ and $d\delta$ quantities with subscripts a, b, and c are independent of the loads, make clear that Δ and $d\Delta$ values may also be determined by superposition, as may be seen from the form of their expressions in Eqs. 7 and 8. Since various determinants are also the same for all loads, the value of dX_a in Eq. 10 may likewise be found by superposition or (which is the same) the expression within the brackets of Eq. 14 for the two sets of loads presents an algebraic sum of the corresponding bracket expressions for the separate loads, which proves the required proposition.

However, the numerical value of dX_a (max) for the two sets of loads is different from the sum of numerical values corresponding to separate loads, because the zero points of the functions under the integral signs in the three cases concerned are generally different.

Two-Legged Frame of Constant Moment of Inertia

The fixed-ended frame shown in Fig. 1, with the height equal to the span and possessing constant moment of inertia, will be investigated for the greatest possible variations in the bending moments at the top end of one of the columns B and at the center of the beam part E, caused by variation in the value of the modulus of elasticity.

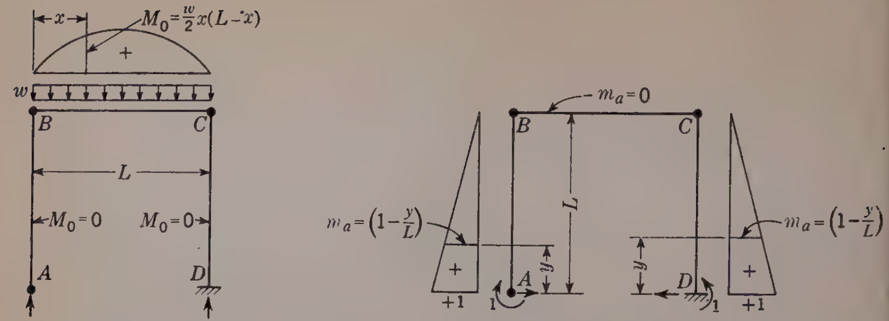
By introducing three hinges at points A, B, and C (Fig. 2(a)), the structure is made statically determinate and the corresponding determinate bending moments M_o are as shown. An unsymmetrical choice of static unknowns X_a, X_b, and X_c, representing the bending moments at the points A, B, and C, respectively, complicates the analysis but provides a means for checking the results, since the expressions for similar quantities at the points B and C will be obtained from equations in a dissimilar manner.

Moments m_a, m_b, and m_c, created in the statically determinate structure by unit moments acting at points A, B, and C, respectively, are shown in Figs. 2(b), 2(c), and 2(d). The sign convention for these moments, as well as for the static unknowns X_a, X_b, and X_c, is such that positive moments produce tension on the inside of the frame.

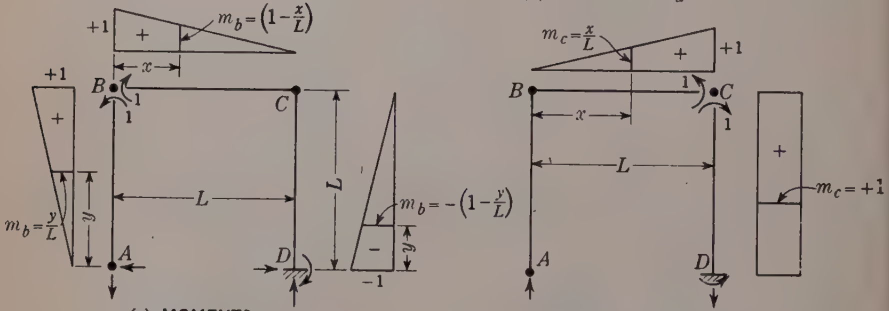
The δ-quantities are now expressed in the integral form and then evaluated. Both kinds of expression will be necessary in further calculation. The integral expressions of δ are composed of three parts, one for each leg of the frame and one for the beam part. In several δ-values, however, some of these integrals vanish because some of the moments M_o, m_a, m_b, or m_c are zero. Thus,

$$\delta_{ao} = \int \frac{M_o\, m_a}{E\,I}\, ds = 0$$

$$\left.\begin{array}{llllll}
 & & \overbrace{\qquad\qquad\qquad\qquad}^{\text{left leg}} & \overbrace{\qquad\qquad\qquad\qquad}^{\text{beam}} & \overbrace{\qquad\qquad\qquad\qquad}^{\text{right leg}} & \\
\delta_{bo} = & \displaystyle\int \frac{M_o\, m_b}{E\,I}\, ds = & 0 & \displaystyle + \int_0^L \frac{w\,L^3}{2\,E\,I}\left(\frac{x}{L} - \frac{2\,x^2}{L^2} + \frac{x^3}{L^3}\right) d\left(\frac{x}{L}\right) + & 0 & \displaystyle = \frac{wL^3}{24\,E\,I} \\
\delta_{co} = & & 0 & \displaystyle + \int_0^L \frac{w\,L^3}{2\,E\,I}\left(\frac{x^2}{L^2} - \frac{x^3}{L^3}\right) d\left(\frac{x}{L}\right) \quad + & 0 & \displaystyle = \frac{w\,L^3}{24\,E\,I} \\
\delta_{aa} = & \displaystyle\int \frac{m_a\, m_a}{E\,I}\, ds = & \displaystyle\int_0^L \frac{L}{E\,I}\left(1 - \frac{2\,y}{L} + \frac{y^2}{L^2}\right) d\left(\frac{y}{L}\right) + & 0 & \displaystyle + \int_0^L \frac{L}{E\,I}\left(1 - \frac{2\,y}{L} + \frac{y^2}{L^2}\right) d\left(\frac{y}{L}\right) & \displaystyle = \frac{2\,L}{3\,E\,I} \\
\delta_{ab} = & & \displaystyle\int_0^L \frac{L}{E\,I}\left(\frac{y}{L} - \frac{y^2}{L^2}\right) d\left(\frac{y}{L}\right) \quad + & 0 & \displaystyle - \int_0^L \frac{L}{E\,I}\left(1 - \frac{2\,y}{L} + \frac{y^2}{L^2}\right) d\left(\frac{y}{L}\right) & \displaystyle = -\frac{L}{6\,E\,I} \\
\delta_{ac} = & & 0 \quad + & 0 & \displaystyle + \int_0^L \frac{L}{E\,I}\left(1 - \frac{y}{L}\right) d\left(\frac{y}{L}\right) & \displaystyle = \frac{L}{2\,E\,I} \\
\delta_{bb} = & & \displaystyle\int_0^L \frac{L}{E\,I}\frac{y^2}{L^2}\, d\left(\frac{y}{L}\right) & \displaystyle + \int_0^L \frac{L}{E\,I}\left(1 - \frac{2\,x}{L} + \frac{x^2}{L^2}\right) d\left(\frac{x}{L}\right) & \displaystyle + \int_0^L \frac{L}{E\,I}\left(1 - \frac{2\,y}{L} + \frac{y^2}{L^2}\right) d\left(\frac{y}{L}\right) & \displaystyle = \frac{L}{E\,I} \\
\delta_{bc} = & & 0 & \displaystyle + \int_0^L \frac{L}{E\,I}\left(\frac{x}{L} - \frac{x^2}{L^2}\right) d\left(\frac{x}{L}\right) & \displaystyle + \int_0^L \frac{L}{E\,I}\left(-1 + \frac{y}{L}\right) d\left(\frac{y}{L}\right) & \displaystyle = -\frac{L}{3\,E\,I} \\
\delta_{cc} = & & 0 & \displaystyle + \int_0^L \frac{L}{E\,I}\left(\frac{x^2}{L^2}\right) d\left(\frac{x}{L}\right) & \displaystyle + \int_0^L \frac{L}{E\,I}\, d\left(\frac{y}{L}\right) & \displaystyle = \frac{4\,L}{3\,E\,I}
\end{array}\right\} \qquad (24)$$


x
$M_0 = \frac{w}{2}x(L-x)$
+
w
B
C
L
$M_0=0$
$M_0=0$
A
D
$m_a=0$
$m_a=\left(1-\frac{y}{L}\right)$
$m_a=\left(1-\frac{y}{L}\right)$
L
y
y
+
+
+1
+1
1
1

(a) MOMENTS M_0 (b) MOMENTS m_a



(c) MOMENTS m_b (d) MOMENTS m_c

FIG. 2.—MOMENT DIAGRAMS; SYMMETRICAL BENT, WITH CONSTANT MOMENT OF INERTIA

Having thus found all δ-values, it is now possible to set up the equations to determine the static unknowns and to compute the determinants required for the evaluation of dX_b (max). Eqs. 1 apply to this case as follows:

$$\left.\begin{aligned} \frac{2L}{3EI}X_a - \frac{L}{6EI}X_b + \frac{L}{2EI}X_c &= 0 \\ -\frac{L}{6EI}X_a + \frac{L}{EI}X_b - \frac{L}{3EI}X_c + \frac{wL^3}{24EI} &= 0 \\ \frac{L}{2EI}X_a - \frac{L}{3EI}X_b + \frac{4L}{3EI}X_c + \frac{wL^3}{24EI} &= 0 \end{aligned}\right\} \quad \text{(25)}$$

The value of the minor determinant corresponding to X_b in the first equation is

$$D_{1b} = \left(-\frac{L}{3EI}\right)\left(\frac{L}{2EI}\right) - \left(-\frac{L}{6EI}\right)\left(\frac{4L}{3EI}\right) = \frac{1}{18}\left(\frac{L}{EI}\right)^2 \quad \text{(26a)}$$

Similarly,

$$D_{2b} = \frac{23}{36}\left(\frac{L}{EI}\right)^2 \quad \text{and} \quad D_{3b} = \frac{5}{36}\left(\frac{L}{EI}\right)^2 \quad \text{(26b)}$$

The major determinant, made up of all the coefficients of the unknowns, is found to be

$$D = \frac{7}{12}\left(\frac{L}{EI}\right)^3 \quad \text{(26c)}$$

On solving the equations the values of the static unknowns come out:

$$\left.\begin{aligned} X_a &= \frac{1}{36}\,w\,L^2 \\ X_b &= -\frac{1}{18}\,w\,L^2 \\ X_c &= -\frac{1}{18}\,w\,L^2 \end{aligned}\right\}\dots\dots\dots\dots\dots\dots\dots\dots(27)$$

the two latter values being equal as might be expected from symmetry.

The next step is to set up the integral expressions for the three functions Δ by Eq. 7:

$$\begin{aligned} \Delta_a = {} & \frac{w\,L^3}{36}\Bigg[\overbrace{\int_0^L \frac{1}{E\,I}\left(1-\frac{2\,y}{L}+\frac{y^2}{L^2}\right)d\left(\frac{y}{L}\right)}^{\text{left leg}} + \overset{\text{beam}}{0} + \overbrace{\int_0^L \frac{1}{E\,I}\left(1-\frac{2\,y}{L}+\frac{y^2}{L^2}\right)d\left(\frac{y}{L}\right)}^{\text{right leg}}\Bigg] \\ & -\frac{w\,L^3}{18}\Bigg[\int_0^L \frac{1}{E\,I}\left(\frac{y}{L}-\frac{y^2}{L^2}\right)d\left(\frac{y}{L}\right) + 0 - \int_0^L \frac{1}{E\,I}\left(1-\frac{2\,y}{L}+\frac{y^2}{L^2}\right)d\left(\frac{y}{L}\right)\Bigg] \\ & -\frac{w\,L^3}{18}\Bigg[\quad 0 \quad + 0 + \int_0^L \frac{1}{E\,I}\left(1-\frac{y}{L}\right)d\left(\frac{y}{L}\right)\Bigg] \\ & + \quad 0 \quad + 0 + \quad 0 \\ & = \frac{w\,L^3}{36}\Bigg[\int_0^L \frac{1}{E\,I}\left(1-\frac{4\,y}{L}+\frac{3\,y^2}{L^2}\right)d\left(\frac{y}{L}\right) + 0 + \int_0^L \frac{1}{E\,I}\left(1-\frac{4\,y}{L}+\frac{3\,y^2}{L^2}\right)d\left(\frac{y}{L}\right)\Bigg] \end{aligned} \quad (28)$$

Within all brackets in Eq. 28 the first integrals refer to the left leg of the frame, the second (which in case of Δ_a are zero) to the beam part, and the third to the right leg. In arriving at the final expression for Δ_a it is the integrals referring to the same part of the frame that are combined, whereas the integrals referring to different parts remain separate. Similarly:

$$\left.\begin{aligned} \Delta_b = {} & \frac{w\,L^3}{36}\Bigg[\overbrace{\int_0^L \frac{1}{E\,I}\left(\frac{y}{L}-\frac{3\,y^2}{L^2}\right)d\left(\frac{y}{L}\right)}^{\text{left leg}} - \overbrace{\int_0^L \frac{1}{E\,I}\left(2-\frac{20\,x}{L}+\frac{36\,x^2}{L^2}-\frac{18\,x^3}{L^3}\right)d\left(\frac{x}{L}\right)}^{\text{beam}} \\ & \qquad - \overbrace{\int_0^L \frac{1}{E\,I}\left(1-\frac{4\,y}{L}+\frac{3\,y^2}{L^2}\right)d\left(\frac{y}{L}\right)}^{\text{right leg}}\Bigg] \\ \Delta_c = {} & \frac{w\,L^3}{36}\Bigg[\overbrace{\qquad\qquad 0 \qquad\qquad}^{\text{left leg}} - \overbrace{\int_0^L \frac{1}{E\,I}\left(\frac{2\,x}{L}-\frac{18\,x^2}{L^2}+\frac{18\,x^3}{L^3}\right)d\left(\frac{x}{L}\right)}^{\text{beam}} \\ & \qquad - \overbrace{\int_0^L \frac{1}{E\,I}\left(-1+\frac{3\,y}{L}\right)d\left(\frac{y}{L}\right)}^{\text{right leg}}\Bigg] \end{aligned}\right\}(29)$$

These expressions for various values of Δ have been checked by evaluating the integrals on the assumption that $E\,I$ is constant and by finding that all Δ's then vanish.

Moment X_b.—The expression for the greatest possible variation of X_b is found by multiplying the integral expressions of various Δ's by their minor determinants, in accordance with Eq. 15, and by recombining the integrals:

$$dX_b\,(\max) = \frac{i}{\frac{7}{12}\left(\frac{L}{E\,I}\right)^3}\left(\frac{L}{E\,I}\right)^2 \frac{w\,L^3}{36\,E\,I}$$

$$\times\Bigg[\overbrace{\int_0^L \left|\frac{1}{18}\left(1-\frac{4\,y}{L}+\frac{3\,y^2}{L^2}\right)+\frac{23}{36}\left(\frac{y}{L}-\frac{3\,y^2}{L^2}\right)+\frac{5}{36}(0)\right|}^{\text{left leg}}$$

$$\overbrace{+\int_0^L \left|\frac{1}{18}(0)+\frac{23}{36}\left(-2+\frac{20\,x}{L}-\frac{36\,x^2}{L^2}+\frac{18\,x^3}{L^3}\right)+\frac{5}{36}\left(-\frac{2\,x}{L}+\frac{18\,x^2}{L^2}-\frac{18\,x^3}{L^3}\right)\right| d\left(\frac{x}{L}\right)}^{\text{beam}}$$

$$+\overbrace{\int_0^L \left|\frac{1}{18}\left(1-\frac{4\,y}{L}+\frac{3\,y^2}{L^2}\right)+\frac{23}{36}\left(-1+\frac{4\,y}{L}-\frac{3\,y^2}{L^2}\right)+\frac{5}{36}\left(1-\frac{3\,y}{L}\right)\right| d\left(\frac{y}{L}\right)}^{\text{right leg}}\Bigg]$$

$$=\frac{w\,L^2}{756}\,i\,\Bigg[\overbrace{\int_0^L \left|2+\frac{15\,y}{L}-\frac{63\,y^2}{L^2}\right| d\left(\frac{y}{L}\right)}^{\text{left leg}}$$

$$\overbrace{+2\int_0^L \left|-23+\frac{225\,x}{L}-\frac{369\,x^2}{L^2}+\frac{162\,x^3}{L^3}\right| d\left(\frac{x}{L}\right)}^{\text{beam}}$$

$$+\overbrace{\int_0^L \left|-16+\frac{69\,y}{L}-\frac{63\,y^2}{L^2}\right| d\left(\frac{y}{L}\right)}^{\text{right leg}}\Bigg] \quad \ldots\ldots\ldots\ldots\ldots (30a)$$

In Eq. 30*a* each integration must be performed with respect to the numerical values of the functions under the integral signs, rather than with respect to the functions themselves, as is indicated by the sign $|\,|$. This necessitates the determination of the points at which the expressions under the integral signs become zero.

Left Leg.—On solving the quadratic it is found that the function under the integral sign becomes zero at the points $y = \frac{1}{3}\,L$ and $y = -\frac{2}{21}\,L$, of which the latter value is outside of the range considered. Therefore:

$$\int_0^L \left|2+\frac{15\,y}{L}-\frac{63\,y^2}{L^2}\right| d\left(\frac{y}{L}\right) = \left|\int_0^{L/3}\left(2+\frac{15\,y}{L}-\frac{63\,y^2}{L^2}\right) d\left(\frac{y}{L}\right)\right|$$

$$+\left|\int_{L/3}^{L}\left(2+\frac{15\,y}{L}-\frac{63\,y^2}{L^2}\right) d\left(\frac{y}{L}\right)\right| = \left|\frac{13}{18}\right| + \left|-\frac{220}{18}\right| = \frac{233}{18} = 12.944.$$

Beam Part.—An approximate solution of the cubic equation in x gives the following values: $x = 0.128\,L$ and $x = 0.874\,L$. Integrating separately in the three intervals between the limits 0, $0.128\,L$, $0.874\,L$, and L, and adding the numerical values of the results:

$$\int_0^L \left|-23+\frac{225\,x}{L}-\frac{369\,x^2}{L^2}+\frac{162\,x^3}{L^3}\right| d\left(\frac{x}{L}\right)$$

$$= |-1.347| + |8.624| + |-0.272| = 10.243.$$

Right Leg.—The integration must be performed separately between the limits $0, \frac{1}{3}\,L, \frac{16}{21}\,L$, and L:

$$\int_0^L \left|-16+\frac{69\,y}{L}-\frac{63\,y^2}{L^2}\right| d\left(\frac{y}{L}\right) = \left|-\frac{41}{18}\right| + \left|\frac{81}{98}\right| + \left|-\frac{925}{882}\right| = 4.153.$$

Summation.—Substituting these numerical values of the integrals in the expression for the required variation (Eq. 30*a*):

$$dX_b\,(\max) = \frac{w\,L^2}{756}\,i\,[12.944 + 2(10.243) + 4.153] = 0.0497\,i\,w\,L^2 = 0.894\,i\,|X_b|\,.\;(30b)$$

in which $X_b = -\frac{1}{18}\,w\,L^2$, as has been found, and 0.894 is the value of the coefficient K expressed by Eq. 16.

Parts of the frame over which the moment of inertia must be increased or decreased in order to cause the greatest possible variation in the numerical value of the bending moment X_b are shown in Fig. 3, the stiffened parts being indicated by heavy lines, and the ones that are made less stiff—by light lines.

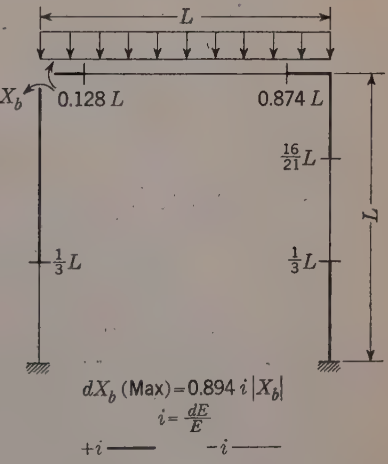


FIG. 3

Moment X_c.—A similar procedure leads to the following expression for the greatest variation in the value of the moment X_c at the top of the right column:

$$dX_c\,(\max) = \frac{w\,L^2}{756}\,i\left[\overbrace{\int_0^L \left|-16+\frac{69\,y}{L}-\frac{63\,y^2}{L^2}\right| d\left(\frac{y}{L}\right)}^{\text{left leg}} + 2\overbrace{\int_0^L\left|-5+\frac{27\,x}{L}+\frac{117\,x^2}{L^2}-\frac{162\,x^3}{L^3}\right|d\left(\frac{x}{L}\right)}^{\text{beam}} + \overbrace{\int_0^L\left|2+\frac{15\,y}{L}-\frac{63\,y^2}{L^2}\right|d\left(\frac{y}{L}\right)}^{\text{right leg}}\right] \quad \ldots\ldots\ldots\ldots(31)$$

Comparing Eq. 31 with Eq. 30*a*, it may be noticed that the first and the third integrals in the former are identical with the third and the first integrals, respectively, in the latter. Furthermore, replacement of the variable x by $(L - x)$ in the second integral of Eq. 31 leads to an expression identical with the second integral in Eq. 30*a*. These relationships between the expressions for dX_b (max) and dX_c (max), of course, are the direct result of the symmetry of points B and C on the symmetrical frame, and they lead to an equality of the numerical values of the variations considered.

Bending Moment at the Center of the Frame.—The greatest variation of the bending moment at the center of the frame E is found by Eq. 23*b*:

$$dM_E\,(\max) = \frac{w\,L^2}{108}\,i\left[\overbrace{\int_0^L\left|-1+\frac{6\,y}{L}-\frac{9\,y^2}{L^2}\right|d\left(\frac{y}{L}\right)}^{\text{left leg}} + 4\overbrace{\int_0^L\left|-1+\frac{9\,x}{L}-\frac{9\,x^2}{L^2}\right|d\left(\frac{x}{L}\right)}^{\text{beam}} + \overbrace{\int_0^L\left|-1+\frac{6\,y}{L}-\frac{9\,y^2}{L^2}\right|d\left(\frac{y}{L}\right)}^{\text{right leg}}\right] \quad .(32)$$

Evaluation of Eq. 32 in the manner illustrated previously leads to:

$$dM_E\,(\max) = 0.0460\,i\,w\,L^2 = 0.663\,i\,M_E \ldots\ldots\ldots\ldots(33a)$$

in which $M_E = \frac{5}{72}\,w\,L^2$.

The numerical value of the maximum variation of the moment at the center of the frame is thus approximately equal to the value of a similar quantity at the column top, but the coefficient K at the center is smaller than at the column top.

It has also been found that

$$dX_a\,(\max) = 0.0447\,i\,w\,L^2 = 1.609\,i\,X_a \ldots\ldots\ldots\ldots(33b)$$

in which $X_a = \frac{w\,L^2}{36}$.

The greatest variation in the moment M_E in the same structure has been determined also for a different load condition—namely, for a vertical concentrated load P acting at the center of the frame. New loading creates new values of δ_{ao}, δ_{bo}, δ_{co} and of the static unknowns. The other δ's and the determinants remain the same. The results are as follows:

$$X_a = \frac{1}{24} P\, L; \qquad X_b = X_c = -\frac{1}{12} P\, L; \qquad M_E = \frac{P\, L}{6};$$

and

$$\begin{aligned} dM_E\,(\max) = \frac{P\,L}{72}\, i \Bigg[& \overbrace{\int_0^L \left| -1 + \frac{6\,y}{L} - \frac{9\,y^2}{L^2} \right| d\left(\frac{y}{L}\right)}^{\text{left leg}} \\ & + \overbrace{\int_0^{L/2} \left| -4 + \frac{24\,x}{L} \right| d\left(\frac{x}{L}\right)}^{\text{beam, left half}} + \overbrace{\int_{L/2}^{L} \left| 20 - \frac{24\,x}{L} \right| d\left(\frac{x}{L}\right)}^{\text{beam, right half}} \\ & + \overbrace{\int_0^L \left| -1 + \frac{6\,y}{L} - \frac{9\,y^2}{L^2} \right| d\left(\frac{y}{L}\right)}^{\text{right leg}} \Bigg] = 0.074\, i\, P\, L = 0.4444\, i\, M_E \quad (34a) \end{aligned}$$

The greatest variation in M_E in the structure loaded with both the uniform and the concentrated loads of the previous problems can be found by direct superposition of the corresponding integral expressions. Assuming, for example, that $P = w\, L$, the following expression is developed for dM_E (max):

$$\begin{aligned} dM_E\,(\max) = \frac{P\,L}{216}\, i \Bigg[& 5 \overbrace{\int_0^L \left| -1 + \frac{6\,y}{L} - \frac{9\,y^2}{L^2} \right| d\left(\frac{y}{L}\right)}^{\text{left leg}} \\ & + 4 \overbrace{\int_0^{L/2} \left| -5 + \frac{36\,x}{L} - \frac{18\,x^2}{L^2} \right| d\left(\frac{x}{L}\right)}^{\text{beam, left half}} + 4 \overbrace{\int_{L/2}^{L} \left| 13 - \frac{18\,x^2}{L^2} \right| d\left(\frac{x}{L}\right)}^{\text{beam, right half}} \\ & + 5 \overbrace{\int_0^L \left| -1 + \frac{6\,y}{L} - \frac{9\,y^2}{L^2} \right| d\left(\frac{y}{L}\right)}^{\text{right leg}} \Bigg] = 0.1196\, i\, P\, L = 0.5067\, i\, M_E \quad (34b) \end{aligned}$$

in which $M_E = \frac{17}{72} P\, L$.

TWO-HINGED FRAME OF VARIABLE MOMENT OF INERTIA

In problems discussed thus far, expressions following the integral signs have been such that the integrations could be performed easily. In more complicated cases integrations must be replaced by summations.

In the uniformly loaded frame shown in Fig. 4 the thickness of the legs changes linearly from 24 in. at the hinge to 36 in. at the top. The thickness of the horizontal member varies from 16 in. at the center to 36 in. at the ends, following the law of a semicubic parabola:

$$h = \frac{4}{3}(1 + 0.01\,x^{3/2}) \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

in which both h and x are expressed in feet. The structure is typical for rigid-frame reinforced-concrete bridges except for the somewhat impractical rectangular outline of the axis. However, this peculiarity of the form, assumed for simplicity of calculation, affects the structural behavior of the frame only slightly. It is required to find the greatest variations of thrust and bending moments at different points of the structure due to variation of the modulus of elasticity. The horizontal thrust H, assumed to act outwardly on the structure, is the only static unknown. If horizontal restraint is removed, the bending moment in the horizontal part of the frame becomes $M_o = \frac{w}{2}(25^2 - x^2)$, and no flexure occurs in the legs. When unit outward thrusts act on the structure, the moments m_a assume the values shown in Fig. 5. The frame is 1 ft wide. Expressions for δ's after simplification are as follows:

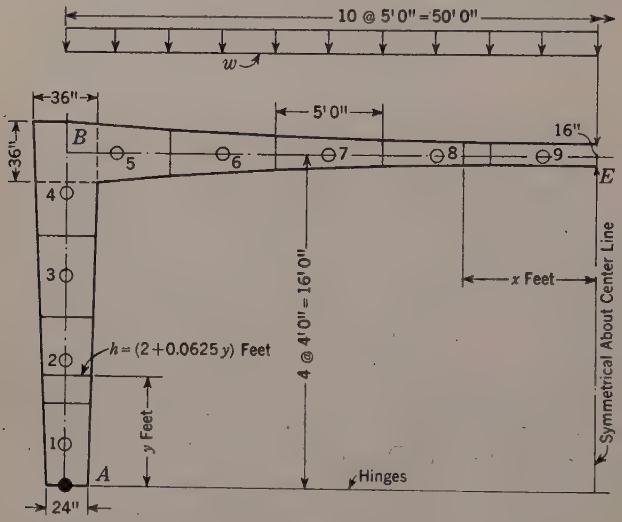


FIG. 4

$$\left.\begin{aligned} \delta_{ao} &= \overset{\text{left leg}}{0} + \overbrace{81\,w \int_0^{25} \frac{(25^2 - x^2)\,dx}{(1 + 0.01\,x^{3/2})^3\,E}}^{\text{beam}} + \overset{\text{right leg}}{0} \\ \delta_{aa} &= \overbrace{2(12)^4 \int_0^{16} \frac{y^2\,dy}{(24 + 0.75\,y)^3\,E}}^{\text{two legs}} + \overbrace{2(6)^4 \int_0^{25} \frac{dx}{(1 + 0.01\,x^{3/2})^3\,E}}^{\text{beam}} \end{aligned}\right\} \ldots (36)$$

For the purpose of summation, the structure is divided into eighteen sections 4 ft and 5 ft long, as shown in Fig. 4. The sectional increments of δ_{ao} and δ_{aa} (that is, the values of the expressions under the integral signs of all sections multiplied by their respective values of dx or dy) are given in Table 1, together with the values of δ_{ao} and δ_{aa} themselves. Then

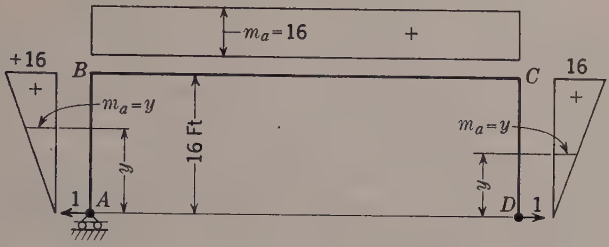


FIG. 5

$$H = -\frac{\delta_{ao}}{\delta_{aa}} = -15.54\,w \ldots\ldots\ldots\ldots\ldots\ldots (37a)$$

The greatest variation in H may be found by Eq. 20*b*, which for finite summation becomes:

$$dH\ (\max) = \frac{i}{\delta_{aa}} \sum \left| \text{Increment of } [\,\delta_{ao} + H\,\delta_{aa}\,] \right| \ldots\ldots\ldots (37b)$$

The values of increments of the expression in brackets are stated in Table 1. All data in this table, as well as the foregoing formulas for δ_{ao}, δ_{aa}, and other values, are given in feet units, and the values in the table refer to the sums of corresponding increments for the pairs of sections located symmetrically with respect to the vertical axis, so that the summation need be extended only over one half of the structure.

TABLE 1.—SOLUTION FOR *dH* (MAX), FIG. 4; VALUES OF THE INCREMENTS IN EQ. 37(*b*) IN FEET

Section	$E\,\delta_{aa}$	$\frac{E}{w}\,\delta_{ao}$	$\frac{E}{w}\,H\,\delta_{aa}$	$\frac{E}{w}\,(\delta_{ao} + H\,\delta_{aa})$
1	40	....	−620	−620
2	258	....	−4,010	−4,010
3	531	....	−8,250	−8,250
4	778	....	−12,300	−12,300
5	1,467	5,450	−22,800	−17,350
6	2,494	24,840	−38,760	−13,920
7	4,311	63,320	−67,180	−3,860
8	7,400	131,520	−115,000	16,520
9	11,537	223,080	−179,290	43,790
Σ	28,816	448,210	−448,210	0
Summation of numerical values of increments....................				120,620

Substituting the numerical values from Table 1, the greatest variation of the thrust proves to be:

$$dH\ (\max) = 4.09\,w\,i$$
$$= 0.263\,i\,|H| \ldots\ldots (37c)$$

In this equation w is expressed in feet units, and so are the various stress quantities to the end of this chapter.

The variations of the moments at the knee and at the crown are found by differentiating the equations of statics relating these moments with H. Calling moment at the knee M_B and at the crown M_E:

$$dM_B\ (\max) = dM_E\ (\max) = 16\,dH\ (\max) = 65.39\,w\,i \ldots\ldots (37d)$$

Since $M_B = -248.65\,w$ and $M_E = 63.85\,w$, dM_E (max) $= 1.024\,i\,M_E$ and dM_B (max) $= 0.263\,i|M_B|$. Fig. 6 shows the parts of the frame that are modified in stiffness for the purpose of creating the greatest variations in H, M_B, and M_E.

Susbtituting a sum for an integral leads to an underestimation of the value of the maximum possible variation of the function. Apart from some error inherent in the substitution of a sum of a finite number of terms for an integral, the value of the greatest possible variation of a function may be substantially the same for integration and summation only if all the zero points of the expression under the integral sign coincide with the boundaries of sections. Since such a condition is not likely to occur, the increment of the sum for the section containing a zero point will be approximately equal to an algebraic sum of the positive and negative areas under the curve in this particular section instead of the sum of the numerical values of the two areas. That means that the value found by summation should be somewhat less than the one found by integration. The difference of the two values, however, is not likely to be great.

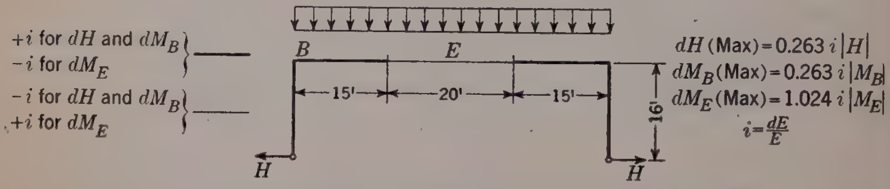


FIG. 6

The uniformly loaded rigid frame of Fig. 4 also has been analyzed, assuming the bottom ends of the columns as fixed. Thus modified, the structure becomes three times statically indeterminate. The following results have been obtained, using feet units: $H = -22.04\,w$; $M_A = 100.00\,w$; $M_B = -252.58\,w$; $M_E = 59.92\,w$; dH (max) $= 0.421\,i|H|$; dM_A (max) $= 1.567\,i\,M_A$; dM_B (max) $= 0.496\,i|M_B|$; and dM_E (max) $= 1.047\,i\,M_E$.

Comparing the fixed-ended frame with the two-hinged frame, it may be seen that dH (max) and dM_B (max) increase considerably when column supports become fixed—increase both in absolute values and in relation to the magnitudes of H and M_B themselves. On the other hand, dM_E (max) remains practically the same.

TWO-HINGED TRUSSED ARCH

As an example of a trussed structure, the two-hinged arch of Fig. 7, uniformly loaded, has been investigated for the maximum possible variations of the thrust and of the stresses in some of the members, caused by variation in E. The cross-sectional areas of all members have been assumed equal. The only static unknown, the thrust, and its greatest variation have been found to be: $H = -135.36\,w$; and dH (max) $= 0.187\,i|H|$, with w in feet units.

The greatest variations of stresses in the truss members have been found by differentiating the expressions for these stresses stated in terms of H. Some of the stresses and their variations are:

Member (Fig. 7)	Stress S	Maximum variation
$L_o L_1$	$-166.77\ w$	$0.187\ i\lvert S\rvert$
$U_4 U_5$	$-103.9\ w$	$1.22\ i\lvert S\rvert$
$U_3 L_4$	$38.6\ w$	$1.21\ i\lvert S\rvert$

In member $L_o L_1$ the stress variation is small compared to the stress itself, whereas in members $U_4 U_5$ and $U_3 L_4$ it is relatively large. In the first two members, uniform load over the entire span represents the most unfavorable condition of loading, and the same is approximately true for the third member considered. In several other members maximum stress variations prove to be even larger in proportion to their respective stresses than in the foregoing three members, but their stresses themselves are relatively small, since the loading considered is by far not the most unfavorable one.

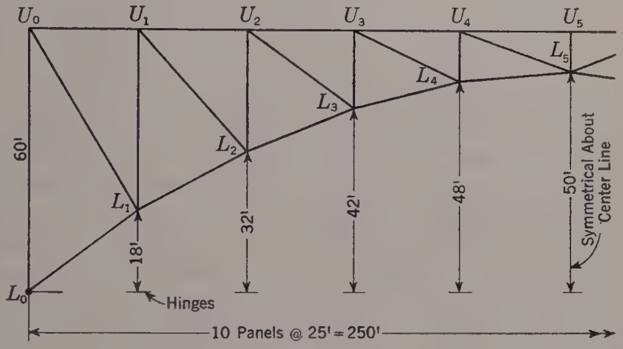


FIG. 7

CONCLUSION

It must be realized clearly that the variation of the static unknown discussed in this paper is an extreme quantity that may be attained only in an exceptional case when the variations of the elastic characteristics causing it assume their maximum possible values (positive or negative), arranged with regard to signs in the most unfavorable and unique manner along the structure. In spite of its improbability, the variation herein considered has a genuine significance as a limiting quantity that the actual deviations of stress functions from their theoretical values cannot exceed, but may approach, in unfavorable cases.

Furthermore, the maximum variation of a stress quantity may be viewed as a measure of the sensitivity of this quantity to small unforeseen and accidental changes in elastic characteristics. Coefficient K in Eq. 16 can be called the coefficient of sensitivity. The higher its value, the more sensitive is the stress quantity to small variations in E, I, or A. Thus, the moment at the crown of the frame of Fig. 4 is much more sensitive to variation in E than the moment at the knee.

From the examples solved it may be seen that the K-values frequently equal unity and occasionally are as high as 1.5 or even 2.0. On the other hand, they are often as low as 0.15 to 0.20. It is possible that there is general increase in sensitivity as the number of static unknowns increases. At least, a comparison of the results for two-hinged and fixed-ended frames suggests this possibility; but the evidence supporting this statement is inconclusive.

In theory, the coefficient of relative variation of the elastic characteristic i is an infinitesimal quantity, but in actual computation it must be given an appropriate finite value, disregarding the error that may result from such assumption.

The numerical value of the greatest variation of a stress quantity in a reinforced-concrete structure is quite considerable. For example, using $dE = 0.2\,E$ and $dI = 0.04\,I$, and making the two corresponding sensitivity coefficients equal to unity, the maximum variation in the stress quantity X proves to be $0.24\,X$. The variations of E and I in this assumption are very conservative, the former being less than the variation observed in the tests at the University of Illinois,[2] at Urbana, and the latter corresponding roughly to a $\pm \frac{1}{8}$-in. error in depth and breadth of a 12-in. by 12-in. section. In this connection it may be stated that the theory presented herein does not take into consideration such other features affecting stress distribution in reinforced-concrete structures as cracking of the sections, validity of Hooke's law, shrinkage, and plastic flow.

In the tests at the University of Illinois some of the moments at column bases differed from theory much more than the 24% found in the foregoing example. The discrepancy, however, was attributed to cracking. In steel structures the greatest variations of stress quantities from the cause herein considered are not likely to be more than 3% to 5% of their values.

The foregoing suggests the futility of using involved methods for finding the "exact" values of stress quantities in most statically indeterminate, reinforced-concrete structures, and it encourages the use of approximate methods. On the other hand, in structures of great importance the determination of maximum values of stress quantities appears insufficient, and must be supplemented by a determination of the greatest variations of these functions.

It is reasonable also to conclude that the legitimate fluctuation of stress values discussed herein should be allowed for by varying working stresses in amounts depending on the sensitivity of the function in question, unless the member is designed on the basis of the theoretical value of the stress computed, modified by the amount of its maximum variation.

APPENDIX

NOTATION

The following letter symbols, used in the paper, conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials, prepared by a Committee of the American Standards Association, with Society representation, and approved by the Association in 1932:[3]

b = breadth of a structural member;
C = constants;
D = a determinant;
E = Young's modulus;
H = a horizontal thrust;
h = height or depth of cross section of a structural member;
I = moment of inertia;
i = a small, theoretically infinitesimal, quantity which is a measure of the greatest variation of an elastic characteristic: $i = \frac{dE}{E}, \frac{dI}{I}$, etc.
K = coefficient representing the maximum percentage by which values of X may change when elastic characteristics change by one per cent;
L = length or height of a structural member;
M = bending moment for structures consisting of flexural members:
  M_E = moment at point E, etc.;
  M_o = moment due to the externally applied loading;
m = moment due to a unit load for members subjected to flexure only:
  m_a = moment due to $X_a = 1$;
  m_b = moment due to $X_b = 1$;
N = direct stress for structures consisting of direct-stress members only;
n = direct stress due to a unit load, for structures consisting of direct-stress members only;
P = a concentrated load;
ds = a small element of a member;
w = a uniform load intensity;
X = a statically unknown force: X_a = force at point a, etc.;
ϵ = an elastic characteristic; and
δ = movement of the point of application of a static unknown in the direction of the unknown.

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[3] ASA—Z10a—1932.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

DESIGN OF SIGN LETTER SIZES

By Adolphus Mitchell,[1] Assoc. M. Am. Soc. C. E., and
T. W. Forbes,[2] Esq.

Synopsis

There has been a growing interest and attention among traffic engineers, and others, in improving the design of highway warning and destination signs. The effectiveness and usefulness of the highway are increased by proper signs just as the proper use of a mechanical article of merchandise is insured by clear and adequate instructions on the box.

There are two characteristics of properly designed highway signs: (1) They must be readable and they must give the motorist sufficient warning to permit making the maneuver comfortably; and (2) they must be designed so as to gain the motorist's attention. To gain attention (characteristic (2)) is a complex problem which deserves separate treatment and is beyond the scope of this paper. Since certain standard color combinations are now agreed upon by state and national highway agencies, these standard color combinations will be assumed in the paper.

Fundamental Factors in Letter Size

On the basis of measurements that have been made of the various factors resulting in the readability, or legibility, of a sign, it is possible to derive methods and formulas for practical determination of the minimum letter size necessary to make them effective under given traffic conditions on the highway.

It is obvious that the size of the letters must depend not only on traffic characteristics (since drivers need more warning at high speeds), but also upon vision because the effectiveness of sign copy depends upon what the driver can see. Knowledge is now available from studies of these various factors to set up a method of the size of letters on highway signs. To compute the size of a letter for a highway sign it is necessary to consider design speed, the location of the sign, warning time, type of letter (letter proportion), and legibility distance.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by May, 1942.

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The following sub-factors influence the determination of warning time: Time for the minimum glance, safety factor in case the sign is not seen at the first possible moment, perception-reaction time, and stopping or deceleration time as necessitated by circumstances.

Design Speed.—Design speed may be defined as that speed at, or below which, a certain proportion of the vehicles operate at the location in question. The proportion (or percentile speed) used by different engineers has ranged from 80% to 90%,[3] and D. W. Loutzenheiser, Jun. Am. Soc. C. E., in 1940, reported[4] an analysis of speed data in an attempt to derive a similar definition for the design application. This report indicates that at least a 90 percentile speed is desirable. Since the higher percentile value results in a higher speed and a longer warning distance, it represents the conservative choice; and the 90 percentile speed of vehicles operating on the highway at the point to be signed is to be recommended for the design of sign-letter size.

Obviously, it is not practicable to make a "spot speed study" at all points along the highway, and since most highways in open country show similar speed characteristics throughout certain sections, it is sufficient to use one speed determination for such sections and another for special conditions, such as suburban and congested urban conditions. At special locations, such as curves, the techniques now in use for speed zoning would be continued. Thus the spot speed check might be replaced by the use of a speed determination from the ball bank indicator, calibrated for such special application.

Sign Location.—The position of the sign on the highway will influence the necessary letter size longitudinally and laterally. In the first place, the sign placed ahead of the hazard will allow the use of a smaller letter size, whereas one at or behind the hazard will require a larger letter size if the driver is to see it at a given distance or warning time away. In the second place, the lateral placement of the sign will influence the point at which the driver's gaze must leave the sign and return to the road. Longitudinal placement is illustrated diagrammatically in Fig. 1. If the driver of vehicle 1 is to read the sign at the position indicated, and if O represents the point of hazard or the point at which the maneuver must begin, a sign placed at a distance A ahead of this point will require a smaller letter size than a sign placed at a position A' behind this point.

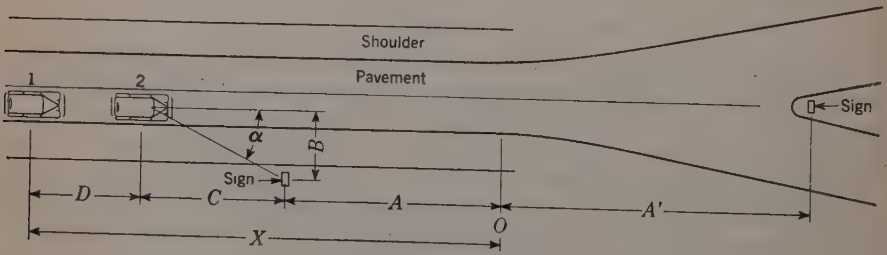

FIG. 1

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[3] "Report of Committee on Safe Approach Speeds," by H. H. Hammond, *Proceedings*, Highway Research Bd., 1940, Vol. 20, p. 653.

[4] "Percentile Speeds on Existing Highways," by D. W. Loutzenheiser, *ibid.*, 1940.

Therefore the distance A may be subtracted or the distance A' added to the necessary warning distance in deriving the necessary legibility distance.

Whether the signs should be placed at position A' or at position A involves the question of attention values, which is beyond the scope of this paper. The lateral position of the sign affects the "cutoff" point of the driver's vision as illustrated in Fig. 1 by the position of vehicle 2. At this point, the line of sight, as the driver is looking at the sign, will form angle α with the center line of the lane of travel as shown. If the maximum allowable angle α is known, the position of vehicle 2 beyond which the driver's line of sight must leave the sign and return to the road can be determined; thus:

$$C = B \cot \alpha \quad \text{(1)}$$

in which B is the distance in feet from the sign to the center line of the traveled lane. The clearest seeing occurs[5] in a small zone in the center of the visual field which is delimited by an optical angle of about 5°. In the area just outside of this 5°, the degree of clear vision reduces rapidly until at about 10° (5° to either side of the line of sight) there is a sharp break in the acuity curve indicating that in a position more lateral than this, the vision experienced in the periphery of the eye is of the fuzzier type. The driver ordinarily glances about the landscape with short quick eye movements, resembling flashes of a highly focused spot light or search light.[6] A practical value for the angle α may be obtained on the basis of allowing two minimum 5° eye movements to the side of the center line of the traveled lane. This will allow some part of the traveled lane to remain in the 10° cone of fairly clear vision up to about 50 ft ahead of the car.

Then, assuming $\alpha = 10°$, Eq. 1 becomes

$$C = B \cot 10° = 5.7\,B \quad \text{(2)}$$

From this relationship, if B is 17.5 ft (one half of the 10-ft lane plus a 10-ft shoulder, plus 2.5 ft from the shoulder to the center of the sign) the distance C will be approximately 100 ft. This gives a rule-of-thumb idea of the magnitudes involved. The hypothetical position of vehicle 2 should be in the lane most unfavorable for vision of the sign in the location in question.

Warning Time.—The importance of thinking in terms of time in order to allow the driver sufficient warning has already been mentioned. Since the process of reading, as well as the perception and reaction of the driver, may be assigned fairly definite time values, and since the use of time values automatically corrects for the greater distance at higher speeds, they are fundamental. If sufficient warning time is given the driver and if he is duly impressed with the nature of the hazard to which he is coming, he will be able to operate comfortably and without any surprise element. To be adequate, warning time must allow for: (1) Minimum time for reading, (2) a safety factor in case the sign is not seen at the first possible instant, (3) perception-reaction time, and (4) stopping or deceleration time necessary for the maneuver involved. Ordinarily, the first three of these time values (those in connection with reading and perception reaction) will pass before any slowing occurs. Therefore, they may be treated as pure time values and converted into distance values by multiplying by the design speed.

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[5] "Textbook of Physiology," by W. H. Howell, W. B. Saunders Co., Philadelphia, Pa., 1940.

[6] *Transactions*, Am. Soc. C. E., Vol. 106 (1940), p. 456.

Minimum Reading Time and Safety Factor.—The shortest possible glance from the road to read the sign and back to the road consumes from 0.6 to 1.0 sec. During this glance the maximum amount of copy which can be read by the ordinary person is from three to four familiar words.[7] This time value results from the fact that it takes approximately 0.2 sec for the eye to stop, focus and read, and another 0.2 sec for the eye to start and move through one of the 5° or 10° jumps. To be conservative, therefore, 1.0 sec is adopted as the time necessary for a single minimum glance from road to sign and back to the road, allowing the shortest possible glance at both the road and the sign.

If more than three familiar words are included in the copy, it has been shown that the time for reading the sign may be increased to as much as from 3 to 11 sec,[8] and such signs, therefore, are impracticable on high-speed highways. Where they are unavoidable the reading time should be increased by one second for each additional three or four familiar words, thus making allowance for the driver to glance back to the road between glances at the sign.

It is still necessary to add a time interval as a safety factor in case the sign is not seen at once. The smallest possible safety factor is one 1-sec glance; that is, the minimum reading time is 2 t_g, in which t_g is the time required for a single glance or 1.0 sec and when signs contain the minimum number of words. When the sign contains more than three words:

$$t_g = \frac{N}{3} + 1.0 \qquad (3)$$

in which N equals the number of familiar words on the sign. If the attention or target value has been properly designed into the sign, the motorist will have seen it before he is able to read it, and Eq. 3, with 2 sec as an absolute minimum, will guarantee him time to read the sign twice unless something distracts him or obstructs his vision.

Perception-Reaction Time.—It takes time for the human nervous and muscular system to react, just as it takes time to complete a call through the telephone exchange. This time value increases as the complexity of the process increases, so that there is a range of time values that may be classified under three headings in order of magnitude—that is, simple brake-reaction time such as that in an emergency situation (from 0.5 to 0.7 sec),[9] perception-reaction time in ordinary traffic situations of medium complexity, and finally, judgment perception-reaction time (2.8 to 3.5 sec).[10]

The perception-reaction time needed to observe signs is intermediate between the simple reaction time and the complex skilled judgment that involves the longest time interval. Interpolated between the values of these two, it will not be far in error to assume a perception-reaction time of 1.5 sec for sign design.

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[7] "A Method for Analysis of the Effectiveness of Highway Signs," by T. W. Forbes, *Journal of Applied Psychology*, Vol. 23, 1939, p. 669.

[8] "Improvement in Highway Safety," by A. R. Lauer, *Proceedings*, Highway Research Bd., Vol. 12, 1932, p. 389.

[9] "Driver Test Results," by H. R. DeSilva and T. W. Forbes, Harvard Traffic Bureau and WPA of Massachusetts, Boston, 1937.

[10] "Methods of Measuring Judgment and Perception Time in Passing on the Highway," by T. W. Forbes, *Proceedings*, Highway Research Bd., Vol. 19, 1939, p. 218.

Deceleration Time.—Consider a point at which some vehicles are decelerating (case 1) while others are continuing with practically no change of speed. This case is best illustrated at grade-separated intersections where some drivers are decelerating and turning off while others are going on through the intersection. Obviously, those drivers who wish to turn off must have the time and distance to decelerate comfortably; and at the same time those who proceed at initial velocity must have sufficient warning to choose the correct lane and to avoid incoming and outgoing vehicles. Somewhat similar conditions hold for intersections at grade on low-volume high-speed highways.

A report by John Beakey,[11] in which the speed checks were made on vehicles approaching a stop sign at a rural intersection, indicates that the general driving public utilized approximately 10 sec on the average for decelerating from 55 miles to 20 miles per hr. If the deceleration times for lower initial speeds as reported in this study are examined, it will be found that deceleration to 20 miles per hr in each case approximated 8 sec. These measurements would seem to justify the use of a value of 8 to 10 sec for comfortable deceleration time on most highways. Since this paper is dealing with the minimum values for necessary warnings, the lower value of 8 sec will be adopted. Deceleration to 20 miles per hr was chosen as being applicable in the majority of highway intersection applications. Time values for higher final speeds are given elsewhere.[11]

Regardless of the speed at which they are operating or the point at which they begin their deceleration, various motorists whose vehicles were checked give themselves about the same time to decelerate. The motorists traveling at 30 miles per hr or less could have decelerated in a shorter time if they had desired. It may be assumed, therefore, that 8 sec represents a comfortable time to the motorist and that where a vehicle is proceeding straight through at ordinary speed, a similar time interval should be allowed for weaving and dodging incoming or outgoing cars. Since the through speed will be V_i (which is assumed as 55 miles per hr), it will always be a higher velocity than the average velocity $\frac{V_i + V_f}{2}$. Therefore, $t = 8$ sec and V_i for determining warning distances will allow sufficient time and distance for both the high speed and the decelerating vehicles.

When all cars must come to a stop (case 2), the stopping distance is the controlling factor, but it must be borne in mind that the value of deceleration used must be such as to allow the motorist 8 sec or more in order to satisfy the time requirement for comfortable operation as derived for the case of simple deceleration without stopping. Transforming Eq. 3,

$$t = \frac{1.47\,(V_i - V_f)}{a} = 8 \text{ sec} \qquad (4)$$

in which V_i is in miles per hour, V_f equals zero, and a is deceleration in feet per second per second.

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[11] "Acceleration and Deceleration Characteristics of Private Passenger Vehicles," by John Beakey, *Proceedings*, Highway Research Bd., Vol. 18, 1938, Fig. 9, p. 81.

MANEUVERING OR DECELERATION DISTANCE

Case 1.—For this case, utilizing the foregoing values for simple deceleration time, the maneuvering distance D is as follows:

$$D = 1.47\,V_i t = 8 \times 1.47\,V_i \qquad (5)$$

in which V_i is the design speed (90 percentile speed of approach and of through vehicles) in miles per hour.

Case 2.—If the distance necessary to stop comfortably results in a corresponding stopping time greater than that shown (case 2), the stopping distance may be used for computing the total warning distance. Deceleration and stopping distance is given by the following familiar formula:[12]

$$s = \frac{(1.47)^2\,(V^2_i - V^2_f)}{2\,a} \qquad (6)$$

in which V_i is initial velocity in miles per hour, V_f is the final velocity at deceleration (which becomes zero for a stopping distance), and a is the deceleration in feet per second per second. The stopping distance thus becomes:

$$s = \frac{1.08\,V^2_i}{a} \qquad (7)$$

To determine comfortable stopping distances for sign-design purposes, it is necessary to choose a value of deceleration a that will fulfil several conditions. It must result in comfortable stopping, must allow the motorist sufficient warning time, and must fulfil the requirements of safe stopping under adverse surface conditions, such as ice, snow, and wet pavements. In other words, the distance allowed the motorist should be adequate under all conditions. Mr. Beakey's study shows a time of 12 sec for a stop from 55 miles per hr, which corresponds to an average deceleration value a of 6.8 ft per sec². However, the actual instantaneous deceleration from which this average value was computed ran as high as approximately 12 ft per sec² as the vehicle approached the actual stopping point. A report by E. E. Wilson[13] in 1940 indicates 8.6 ft per sec² to represent a comfortable (average) deceleration as reported by drivers and passengers in stops from 50, 60, and 70 miles per hr. Thus, from 6 to 8 ft per sec² as an average deceleration apparently represents the comfortable deceleration range chosen by motorists. Maximum instantaneous values will be much higher, of course, but this does not concern the present application.

It is recommended that an average deceleration value of 4 ft per sec² be used for designing sign letters. This will maintain the maneuvering time in the 8-sec range indicated by Mr. Beakey's study for the lower speeds. For instance, in computing stopping distance from 30 miles per hr, 8 ft per sec² gives $t = \frac{1.47 \times 30}{8} = 5.5$ sec, which is an inadequate time value. The deceleration of 4 ft per sec² raises this value to 11 sec, which is adequate. Furthermore, since this value corresponds to a coefficient of friction of about $f = 0.12$, it should be adequate for icy conditions.[12] Therefore, the sign design value recommended is $a = 4$ ft per sec².

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[12] "Skidding Characteristics of Automobile Tires," by R. A. Moyer, *Bulletin No. 120*, Iowa Eng. Experiment Station, 1934.

[13] "Deceleration Distances for High Speed Vehicles," by E. E. Wilson, *Proceedings*, Highway Research Bd., Vol. 20, 1940, p. 393.

WARNING DISTANCE

Having now determined all the components of warning time and warning distance and explained the basis upon which they have been derived, it is possible to assemble the factors to yield the total warning time and warning distance, in Figs. 1 and 2.

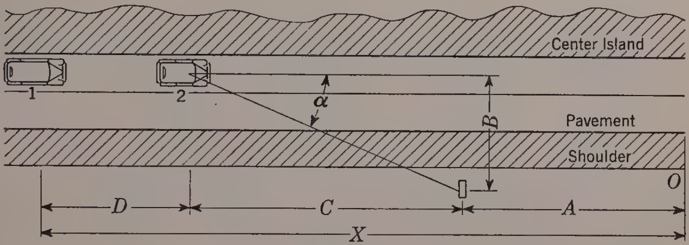

FIG. 2

Case 1.—For case 1 the total warning time T is given by

$$T = 2\,t_g + t_p + t_d \qquad (8)$$

in which t_g is the time for a single glance at a sign, t_p is perception-reaction time, and t_d is deceleration time. Assuming phrases of not more than three familiar words and substituting the value derived previously, this becomes $T = 2.0 + 1.5 + 8.0 = 11.5$ sec. When the sign contains a greater number of words, the first term must be increased accordingly.

The total warning distance then becomes

$$X = 1.47\,V_i\,T = 1.47 \times 11.5\,V_i \qquad (9)$$

in which V_i is the sign-design speed of the highway, and T is the total warning time. For convenience, it is suggested that the time value be rounded off to 11 sec, giving $X = 16.2 \times V_i$ in miles per hr, or $X = 11 \times V_i$ in ft per sec. This will yield a warning distance in which the sign placement and design should allow the sign to be read at 810 ft for a design speed of 50 miles per hr. Other values are given in Table 1.

TABLE 1.—TYPICAL WARNING DISTANCES, X, IN FEET[a]

Deceleration distances required to:	VELOCITY, V_i, IN MILES PER HOUR			
	30	40	50	60
Enter an intersection at 20 miles per hr	486	648	810	972
Stop at an intersection	393	638	932	1,281

[a] Level grades have been assumed throughout.

Case 2.—When all cars must stop at or before the hazard, the total warning distance may be computed as follows:

$$D = 1.47\, V_i\, (2\, t_g + t_p) + s \dots\dots (10)$$

in which s is the stopping distance defined by Eq. 7. Substituting the values derived in the foregoing sections:

$$X = 1.47\, V_i \times (2.0 + 1.5) + \frac{1.08\, V^2_i}{a} = 1.47\, V_i \times 3.5 + \frac{1.08\, V^2_i}{4} \dots (11)$$

in which V_i is initial or design speed in miles per hour. As an example, a stop sign, designed for a speed of 50 miles per hr, thus requires a total warning distance of $X = 257 + 675 = 932$ ft (see Table 1).

CHOICE OF LETTER SIZE

Having determined the total necessary warning distance X (see Figs. 1 and 2) and knowing the distance A from the hazard, at which the sign is to be installed, the necessary legibility distance L of the sign is

$$L = X - A = C + D \dots\dots (12)$$

Legibility distances for letters of different widths[14] have been determined in an extended series of tests using from 100 to 250 different people as observers.[15] The findings for daylight conditions may be briefly summarized as follows:

U. S. standard series	Width	Letter legibility, l, in feet per inch of letter height
B	Narrow	33
C	Medium	42.5
D	Wide	50

Dividing the necessary legibility distance (L) by the appropriate letter legibility (l) gives the letter size indicated; thus:

$$\frac{L}{l} = H \dots\dots (13)$$

in which H is the letter height of that particular design letter needed.

Again illustrating, if $X = 932$ ft, $A = 400$ ft, and a wide letter is to be used: $H = \frac{932 - 400}{50} = \frac{532}{50} = 10.6$ in., and a 12-in. letter is indicated. Since it is quite possible to locate a sign so far in advance of the hazard that this computation would call for an absurdly small letter height, the position chosen for locating the sign must be checked. The distance D must be great enough to allow at least minimum reading time (see Eq. 3). Therefore, from Fig. 2,

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[14] "Manual on Uniform Traffic Control Devices for Streets and Highways," A.A.S.H.O. and National Conference on Street and Highway Safety, Washington, D. C., 1937.

[15] "Legibility Distances of Highway Destination Signs in Relation to Letter Height, Letter Width, and Reflectorization," by T. W. Forbes and R. S. Holmes, *Proceedings*, Highway Research Bd., 1939, Vol. 19, p. 321.

$$D = X - A - C = X - A - \text{cotan } 10° \ldots\ldots\ldots\ldots (14)$$

For a sign with three words or less, $D \geqq 1.47\ V_i \times 2\ t_g$.

Case 1.—When some cars proceed, some decelerate, but none must stop, the design speed is determined first by computing X from Eq. 9, or reading it from Table 1. If the sign is to be 400 ft ahead of the hazard ($A = 400$) the necessary legibility distance $L = X - 400$ ft. Next obtain the legibility per inch of letter height (l) from the text (see heading "Choice of Letter Size").[15] The letter height $H = \frac{L}{l} = \frac{X - 400}{l}$. To check for sufficient reading time, obtain C by Eq. 2. Then the following condition must hold (see Eq. 13): $D = L - C \geqq 1.47\ V_i \times 2\ t_g$.

For 3-word or 4-word signs this becomes $1.47\ V_i \times 2$. For more words, t_g should be increased as shown in Eq. 3.

Case 2.—Where all vehicles must stop, the design speed is determined as before; the total warning distance X is computed from Eq. 11 or Table 1; and A or A' is subtracted or added, respectively. Again suppose it is desired to place the sign 400 ft ahead of the hazard; then $L = X - 400$ and the letter height $H = \frac{X - 400}{l}$, in which l is legibility distance of the letters to be used.[15] Finally, the designer should check for sufficient reading time as in case 1. In both cases, if the reading time is too short, determine A, or the sign location, by reversing the last few steps. Obtain D from Eq. 14, C from Eq. 2, and $A = X - (C + D)$. The sign must be mounted at the position indicated. Finally, compute L and H for the new sign position by the appropriate method for either case 1 or case 2.

PRACTICAL CONSIDERATIONS

Obviously certain practical decisions as to where the sign is to be placed, whether both pre-warning and a sign at the hazard or intersection are necessary, and similar questions must be answered before the necessary letter height can be determined. Furthermore, in choosing which of the standard alphabet series to use for the sign copy the designer must take into consideration such factors as the length of place names involved and the general shape and limiting dimensions allowable for his sign. In certain cases where practical requirements make it necessary to include more than three words in a given sign, it has been found desirable to use two different letter sizes in the copy. A larger size has been used for certain important words intended to be read at the greatest distance and a smaller size for the less important legend intended to be read at a closer position and with a second glance of the driver's eye.[16]

SUMMARY

A method has been described for determining the necessary letter size for effective highway sign design in terms of warning time, warning distance, and a

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[16] Sign Specifications of the Pennsylvania Turnpike Commission, Missouri State Highway Dept., and others.

90 percentile design speed. The increased warning distance necessary at higher speeds has been provided through the use of driver warning time as the basic consideration. Appropriate braking distances have been introduced where necessary.

Table 1 (or Eqs. 9 and 11) furnishes values for convenient computation of the letter height needed, and a simplified outline of procedure is given in the paper.

By use of such a method, it is possible to fit the sign to the highway and the driver and to obtain consistently effective signs for widely varying conditions of velocity and sign location with respect to the hazard or the maneuver point.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

PROTECTIVE AND REMEDIAL MEASURES FOR SANITARY AND PUBLIC HEALTH ENGINEERING SERVICES

PROGRESS REPORT OF THE SANITARY AND PUBLIC HEALTH ENGINEERING DIVISION OF THE NATIONAL COMMITTEE OF THE SOCIETY, ON CIVILIAN PROTECTION IN WAR TIME

1. Introduction

This Report is issued to present certain engineering aspects of civilian protection in wartime in the field of Sanitary and Public Health Engineering. Its preparation is the result of the cooperative effort of many Subcommittees, representing the local sections of the Society. A list of the Chairmen of these Subcommittees appears at the end of the Report. Those interested in an explanation of the Report, or those who are of general assistance in the areas represented by the different Subcommittees, are invited to seek their cooperation.

The measures required for civilian protection in wartime in the field of Sanitary and Public Health Engineering are directed against attack from the air and from gun fire and against sabotage. However, it should be noted that many problems in this field will result from large emergency concentrations of population in areas which do not have the peacetime facilities for serving such large sudden increases in the population. Therefore, the methods, plans, and procedures to be developed should include preparation for emergency population shifts. The construction and operation of sanitary utilities for large military establishments, including ordnance work, also should be considered, as the adequacy of Sanitary and Public Health Engineering services in such areas is highly important.

In wartime, supplies and men are used first and to the utmost for military purposes. Those in charge of sanitary and public health engineering utilities

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Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **May, 1942.**

may be handicapped by this condition. Careful planning and devoted effort are all the more necessary.

2. Attack

The methods of attack for which civilian protection is required are from the air and from gun fire. A good description of aerial attack appears in "Civilian Defense Protective Construction—Structure Series, Bulletin No. 1," prepared and published by the U. S. War Department.

Attack from gun fire has not been described, to our knowledge, as applied to civilian protection. The reason for this probably is that communities subject to gun fire are in the active war zone and the entire civilian protection is subordinated to military activities.

3. Sabotage

It is probable that the amount of sabotage will increase under wartime conditions. Already, many communities have taken precautions against attack by saboteurs on sanitary engineering facilities. Such attack includes destruction by explosives, the introduction of poisons into water, milk, and food supplies, thefts of essential parts of equipment, and tampering not only with equipment and structures, but also with the morale and the reliability of employees.

4. Objectives and Scope

This Report relates to the following principal public structures and services: (*a*) Water supply, (*b*) sewerage, (*c*) refuse disposal, (*d*) street cleaning, (*e*) public comfort stations, (*f*) milk supplies, (*g*) rooming houses, and (*h*) restaurants. These terms are broadly inclusive of the many parts of each item.

In general, the objective of the Report is to "create a source of engineering information and to develop engineering ideas into a new technology" applicable to civilian protection in wartime in the fields of Sanitary and Public Health Engineering. It is hoped that the Report can be placed in the hands of responsible public officials in each vulnerable community.

5. General Criteria

Special measures for civilian protection should be directed toward structures and services for which they are most needed. Some structures and services will require comparatively little attention in the engineering field. The following general criteria are suggested:

(*a*) First, the likelihood of being sabotaged or attacked;

(*b*) Second, the extent of damage that might be accomplished by sabotage or bombing, including the interruption of important civilian defense and war activities, the general well-being and health of citizens, and the effect upon the morale of the people; and

(*c*) The difficulty of making repairs, and the time required.

6. Blackouts

Blackouts have generally been considered as a good method of lowering the effectiveness of aerial attack and plans for blackouts have been or will be described in other bulletins. Thus, emergency repair work often will need to be done in the dark, and preparation should be made for it. During blackouts it is essential that power be available for operating pumps and other important pieces of equipment.

7. Protective Lighting

Protective lighting of structures has been given much study in the United States, but is still in a developmental stage. Therefore, municipal officials should look for the results of further investigation and experiment. In the long run, the conclusions regarding effective blackouts will have some bearing on the matter of protective lighting. In general, protective lighting is considered a desirable method of protection against sabotage. S. G. Hibben, Director of Applied Lighting of the Westinghouse Electric and Manufacturing Company, summarizes some of the cardinal problems of protective lighting, as follows:

> (*a*) In general, wiring systems should be underground for protection and should be flexible, so that some of the protective lighting can be temporarily extinguished during an air attack, or limited to lighting that would not disclose the details or exact position of key structures. The wiring should be controlled by a plurality of switches, and circuits should be split up so that power for connection failures would not remove solid blocks of lighting.
>
> (*b*) The lighting units should be numerous and of relatively small size, so that the breaking or burning out of a lamp will not cause a major loss of light. The lighting units should be nonfragile and weatherproof.
>
> (*c*) Low mounted projectors on or back of fence lines, pointing away from key structures, are desirable for yard lighting, including Fresnel lenses, or reflector bulbs, such as Mazda 150-watt lamps. These should be placed so that glare will prevent a person outside the fence from seeing objects within the enclosure. High mounted floodlights on poles do not at the present time appear to be as desirable as the low mounted projectors. If floodlights are used, they should be arranged so as to leave unlighted stretches for the use of watchmen.
>
> (*d*) Flood lighting of buildings should be avoided, since an attacking airplane makes its observations from an angle of about 45°, rather than straight downward.
>
> (*e*) Reflecting surfaces should be avoided, such as water; and, to avoid reflection, protective lighting should be kept away from concrete surfaces likely to become wet.
>
> (*f*) So far as possible, the lighting units should be shielded from direct upward vision, or designed for a sharp vertical cutoff. Fresnel lens control with metal hoods or visors is suggested, and it is possible that some of the special projector automobile head lamps may be adapted to protective lighting. Colored lighting, such as that from sodium or mercury lamps, is undesirable.

It has been suggested that access to important properties during blackouts may be protected by an alarm system on a protective fence, to indicate the location of an attempted intrusion by a saboteur.

Thus, blackouts and protective lighting are related and are both in a developmental stage in regard to civilian protection in wartime. Further research is desired, and those responsible for civilian protection should be alert for further conclusions.

8. *Fence Protection.*—Various types of fence protection have been used effectively, as, for example, barbed-wire stockades with and without charged wiring, and fencing with "electric eye" and other automatic lighting and signal devices. (A very effective and economical wiring device for fences, being developed by the New England Power Company, is quite different from the "electric eye" theory and is based on the principle of radio-activity. It can be arranged to throw a switch for illumination or other signal warning when any one approaches within 6 ft of the fence.)

PART *A*.—WATER WORKS

The most important of the Sanitary and Public Health Engineering services is the water works. The following paragraphs outline the major parts, the troubles that may result from bombing, attack, or sabotage, and protective and remedial measures. Fig. 1 is an outline of a typical water works.

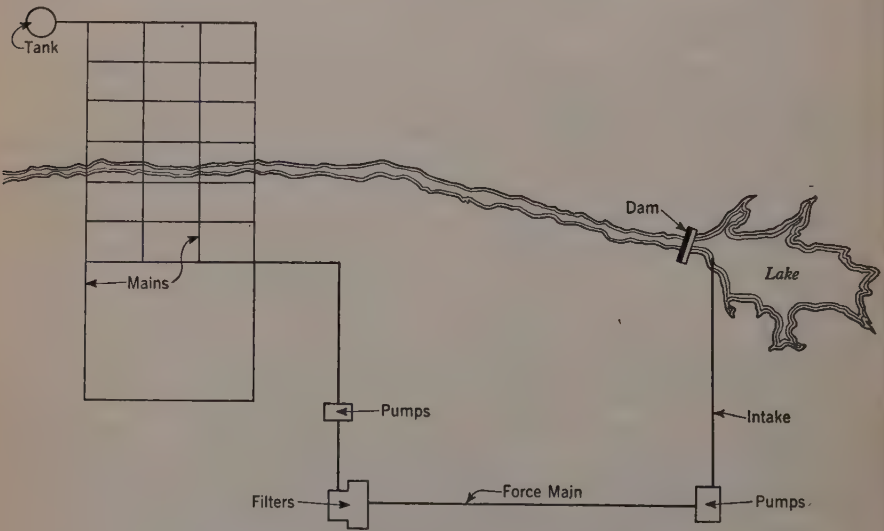

Fig. 1.—Typical Water Supply System

9. Sources of Supply

Water supply is either from a surface or an underground source. Surface supplies are from natural lakes, impounded reservoirs on water courses, and large rivers not requiring storage. Underground sources, in general, are from wells, but occasionally from infiltration galleries. For each of these sources, there are critical elements, damage to which might put the entire water works out of service. Thus, in a natural lake, possible troubles comprise malicious pollution or poisoning, and destruction of the intake works by sabotage or attack. Protective and remedial measures are as follows:

Policing of vicinity and enactment of more strict ordinances governing the use of a watershed.

Regular analyses adapted to detect poisons and unexpected pollution.

Watchman and other precautions against sabotage.

Crews and equipment experienced and ready for repair of the intake works.

Use of an emergency intake or alternate source of water.

The critical elements of an impounded water course or artificial lake are much the same as those of a natural lake, and the protective and remedial measures are similar, so far as they apply. However, the poisoning or pollution of a river source by saboteurs is easier and more likely to be successful than that of a natural lake source of water. Thus, rigorous policing of rivers, so far as possible, is desirable, but principal reliance should be made on chemical and bacteriological analyses made at shorter intervals than required in peacetime. In an impounded supply, there is also danger to the impounding works or dam. Special comments relating to the protection of dams are stated in a later paragraph. The crews and equipment for repair should be experienced, not only in work on intake structures, but should also be prepared for the repair of the impounding structures.

The critical elements of a supply from a large river are much the same as those of an impounded supply, and the protective and remedial measures are similar. As a river not requiring storage is likely to have a relatively large flow of water, the dilution of malicious poisons and pollution will be large and this danger will be correspondingly reduced. Materials and equipment should be available for the prompt and quick use of an emergency intake, which should be available.

The critical elements of an underground source of water are pollution and poisoning by sabotage, and destruction, either by sabotage or attack. It is obviously advisable to have available more than one well to reduce the probability of a complete loss of water. If the supply comprises only one well, arrangements should be made for additional wells in different locations, not likely to be damaged by a single explosion. Otherwise, arrangements should be made for an emergency supply from some other source of water, with provision for temporary chlorination.

Protective coverings can be built over wells and other structures on land, which appear to be highly desirable for critical and vulnerable structures. These are described subsequently in Paragraph 18.

10. Structures for Delivery of Water

Pumping stations comprise the principal structures for the delivery of water. In some cases, a low lift pumping station is required to deliver a surface water to a water treatment plant. In general, there is a high lift or high service pumping station. For small and moderate size cities, one pumping station is generally sufficient, whereas larger cities often have several. For well supplies, there is usually one pump for each well.

If there is a single source of supply and one pumping station, this structure is one of the most critical in a water works. If the station is put out of commission by sabotage or attack, the water supply is likely to be cut off until repairs can be made. Therefore, policing against sabotage is of great importance. Protective and remedial measures embody, in the first place, duplicate and spare units and parts. Steam power pumping stations, in particular, should have duplicate units and these might well be electrically driven, or driven by gasoline or oil engines. Where it is possible, such emergency or stand-by units should be fully protected, preferably by locating them underground at some distance from the main pumping station. Electric pumps may be made ineffective by damage to a power station at some distance, or to transmission lines. Thus, stand-by or emergency pumping units should be made available. Spare parts should include valves, pipes, fittings, pumps, motors, and transformers. These should be stored in a separate location at a distance from the pumping station, to avoid loss of the replacements through one explosion. Because of the likely complicated piping and connections at pumping stations, advance planning of the emergency reconstruction should be careful and complete.

Often, a single supply main delivers the water from a source to a pumping station or water treatment plant. These mains are usually underground and thus are not visible from the air. Damage to such structures is more likely to be accidental or by sabotage. A break of a single supply line will result in putting the water works out of commission. Protective and remedial measures include policing and suppression of sabotage. The most effective protection comprises an emergency source of water, and it is desirable that such an emergency supply be provided where there is sole reliance on a single long supply main. Important valve structures can be given a protective covering. Spare lengths of pipe should be available, with couplings and sleeves for quick repair, and these should be stored at a convenient isolated location. Crews and equipment for repairs should include pumps, excavating equipment, and the like, suitable to the construction problems likely to occur along the supply main. A careful record should be available of the location of all cutoff valves and drains, with provision for their location at night, so that the broken part of the supply main can be quickly isolated.

11. DISTRIBUTION SYSTEMS

The distribution system begins with the pipe lines extending from the main valve chamber at the high service pumping station, and includes the grid work of distribution mains throughout the city. The distribution system is highly vulnerable to air attack. It has been found from British experience that, in vulnerable communities, the distribution system may have broken water mains and may be damaged by bombs as frequently as once for each mile of distribution system under a series of attacks. The breaking of a water main causes much collateral damage. It reduces the effectiveness of fire protection until the break is repaired. There is likely to be a considerable amount of erosion before water is cut off, thus damaging other underground service conduits, as well as pavements and adjacent foundations. Speed is one of the essential factors and this requires a prompt shut-off of the water and a rapid repair. Therefore, one of the essential requirements is a complete map and record of the distribution system, including the location and type of valves. Copies of such records and maps should be guarded carefully, but should be in the hands of the personnel charged with the responsibility of handling the situation and making the repair. British experience indicates that debris is likely to be driven several hundred feet into the main from the crater caused by the explosion. Thus, facilities for removing such debris are important. These conditions may easily contaminate the interior of the mains, and accordingly facilities must be available for chlorination at the site of the repairs. Obviously, the most important mains are those from the pumping station to the distribution grid; and the next in importance are the larger feeder mains. The procedure for repairing a broken water main is described in a later paragraph. Auxiliary personnel should be trained under the direction of the water department and should be available for prompt mobilization. A complete set of emergency supplies for making repairs should be on hand, carefully stored. In large cities, several sets of emergency supplies should be available. A partial list includes the following items:

Trucks
Cleaning rods and equipment
Chlorinating supplies
Portable power pumps with suction and discharge hose
Portable hand pumps
Portable lights
Wooden and perhaps steel sheeting and other lumber
Trench jacks and braces
Paving breakers
Gasoline engine driven compressors
Valves
Rubber boots, coats, hats, and gloves
Hardware, including nails, bolts, and nuts
Valve boxes
Sand bags
Brick, cement, sand, and gravel
Warning signs
Dynamite, blasting caps, and fuses, where rock may be encountered
Pipe cleaning equipment
Shovels
Picks
Bars
Trowels
Hoes
Axes
Drills, hammers, mauls
Rope with hook attached
Scrapers
Saws
Derricks
Chain and falls
Pipe and fittings
Pails
Lanterns and batteries
Canvas
Wheelbarrows
Rope
Wire cable
Chains
Grease, oil, gasoline, and kerosene

12. *Water and Sanitation Facilities.*—It will be necessary to provide water and sanitation facilities in the area affected by the broken main. Filter units mounted on trucks are available, which can draw water from a roadside

ditch or elsewhere, and filter and sterilize it for drinking use. However, it will be necessary to instruct and warn the people living within the affected area, and an organization is necessary for this purpose. Announcements should be made, and signs should be available, warning people regarding the use of the water until it is safe, and giving instructions as to the location of sanitary facilities.

13. *Exposed Piping and Conduits.*—Piping and conduits on bridges or in other exposed places may be particularly vulnerable to sabotage and should receive protection.

14. WATER TREATMENT PLANTS

In general, water treatment plants are those to provide a safe drinking water by filtration and sterilization, or to provide an improved water by removing color and organic matter or some of the hardening and other mineral constituents. The extent of damage may be sufficient to put the entire treatment process out of use for several weeks or months; or it may be minor. An emergency or alternate source of supply should be available, even if limited to a sufficient quantity for drinking purposes distributed by tank trucks. In some cases, the service can be restored promptly, if arrangements have been made for by-passing the treatment works and delivering adequately chlorinated raw water to the distribution system. For this purpose, spare chlorinating equipment should be available stored at a distance from the treatment plant. For a short period, the water can be made safe by boiling at each place of consumption, but long periods of boiling water tend to become a hardship and result in the relaxation of this safeguard.

Community-wide distribution of bactericidal compounds and solutions for the use of individuals might be a safer and more practicable solution to the problem. This could be accomplished by preparing, for distribution among the public, capsules or vials, each containing a sufficient dosage for disinfection of a certain practical quantity of water, such as one to five gallons.

TABLE 1.—CHLORINE REQUIRED FOR WATER TREATMENT

Item No.	Treatment (1)	Available chlorine (%) (2)	GRAINS[a] OF CHEMCAL REQUIRED TO TREAT:		
			5 gal (3)	1 gal (4)	1 gal (disinfectant)[b] (5)
1	Calcium hypochlorite	65	1.2	....	87.5
2	Chlorinated lime	25	3.5	0.7	232.
3	Sodium hypochlorite solution[a]	5	1.1[a]	5[a]	3.2[a]

[a] Units are grains (1 scruple = 20 grains) except in Item 3 where they are, respectively: 1.1 cc, 5 drops, and 3.2 oz. [b] Quantities to be added to 1 gal water to make a disinfecting solution containing 0.1% available chlorine. For a dosage of 3 ppm, add 1.9 oz of any solution prepared according to values in Col. 5 to 1 gal of the water to be disinfected, or 9.3 oz to 5 gal of water.

As an example, the required amounts of various chlorine compounds for such preparations are shown in Table 1. If it is desired to add the commercial preparation directly to the water to be disinfected, the amounts given in Cols. 3 and 4 are sufficient to provide a dosage of 3 ppm.

It may be more convenient to make a 0.1% available chlorine solution of the original commercial product and use this in the disinfection as indicated in Col. 5.

15. Elevated Storage Tanks

Elevated tanks are subject to destruction by sabotage or attack. They should be policed and protected as a routine matter. Such storage is a valuable reserve for fire protection, if the service is disrupted by damage elsewhere. Valves should be available adjacent to the elevated tanks and their location recorded and known for immediate shut-off, if the tank is destroyed. Elevated tanks may be a good target for aerial attack, and some method of camouflage should be developed.

16. Fire Protection

The water supply of a community is important, not only for drinking uses and for general sanitation, but also for fire protection. In the case of attack, fire protection is especially important. Therefore, the ability to make rapid repairs to damaged structures and pipes is vital. Fire fighting procedures prior to attack and in wartime have been described in other bulletins and are therefore not included (see, for example, Items *O*1 and *O*9 in the Appendix). However, the ability to furnish water under pressure is the first essential of fire protection. This is a further emphasis of the need of careful planning and organization to maintain the water supply service.

TABLE 2.—Required Fire Flow, Fire Reserve, and Hydrant Spacing, as Recommended by the National Board of Fire Underwriters

Population (thousands)	Fire Flows		Fire Reserve (million gal)	Area per Hydrant (Thousand Sq Ft)	
	Thousand gal per min	mgd		Engine streams	Hydrant streams
1	1.0	1.5	0.3	120	100
2	1.5	2.0	0.4	. .	90
4	2.0	3.0	1.2	110	85
6	2.5	3.5	1.5	. .	78
10	3.0	4.5	1.9	100	70
13	3.5	5.0	2.1	. .	. .
17	4.0	6.0	2.4	90	55
22	4.5	6.5	2.7	. .	. .
28	5.0	7.5	3.0	85	40[b]
40	6.0	8.5	3.6	80	. .
60	7.0	10.5	4.4	70	. .
80	8.0	12.0	5.0	60	. .
100	9.0	13.0	5.5	55	. .
125	10.0	14.5	6.1	48	. .
150	11.0	16.0	6.7	43	. .
200[a]	12.0	18.5	7.6	40	. .

[a] For populations greater than 200,000 and for local concentrations of streams, see outline, National Board of Fire Underwriters.

[b] For fire flows of 5,000 gal per min or more.

Requirements of water for fire protection may be greater during wartime than otherwise. Therefore, at least the usual requirements for fire protection in peacetime should be available. The requirements of the National Board of Fire Underwriters regarding water and hydrants for fire protection service are shown in Table 2. Communities should survey their water distribution systems and ascertain what additions should be made, if any, to meet reasonable peacetime requirements, as a preparation for wartime conditions. This survey should also include a study of the necessary valves to permit the use of adjacent parts of the distribution system not damaged by aerial attack or explosion.

Small quantities of water stored in tanks, vats, and cellars, and widely distributed throughout the city have been found to be of great value in London for temporary fire protection while repairs are being made.

17. Repair of Water Mains

Water mains may be broken frequently by aerial attack and sabotage. As a broken water main disrupts sanitary service and fire protection, the quick repair of the main is essential. Experience in England has indicated that lead joints are not as satisfactory for this purpose as so-called "mechanical" joints. Therefore, a careful study has been made of the type of mechanical joint best suited to this purpose. British experience indicates that the time of making a repair may be reduced, by careful attention to such details, from some 7 hr to less than 2 hr. This is a very important gain. The principal difficulties in repairing a broken water main may be summarized as follows:

(*a*) There will probably be water in the trench where the break occurs and dewatering to permit making a lead joint will delay the repair.

(*b*) The ends of the broken main will be jagged. The repair fittings should be such as not to require the cutting off of the jagged ends to the extent that would be required to fit a standard bell.

(*c*) There may be a considerable uncertainty as to the actual outside diameter of the broken main. Therefore, the fitting to be used should have as much margin in its inside diameter as can be afforded by a mechanical joint. The outside diameters of a number of typical water mains are shown in Table 3.

(*d*) The subsoil under the main may be loosened by the explosion and it may be necessary to construct the new water main around the edge of the crater. This is not as desirable as a straight run of pipe, but is indicated as a possible necessary alternative.

TABLE 3.—Outside Pipe Diameters; 3 In. to 48 In.
(All Dimensions in Inches)

Nominal inside diameter	Cast-Iron Pipe; Classes:				Steel Pipe; Thicknesses in Sixteenths of an Inch:					Transite Pipe; Classes:			
	A	B	C	D	4	5	6	8	10	50	100	150	200
3	3.78	3.84	3.90	3.96	3.5	....	....	....	....	3.66	3.70	3.88	4.20
4	4.84	4.90	4.96	5.04	4.5	....	....	....	....	4.66	4.70	4.85	5.20
6	6.88	6.96	7.02	7.10	....	6.62	....	....	....	6.72	6.76	6.95	7.50
8	8.92	9.02	9.12	9.20	....	8.62	....	....	....	8.84	8.88	9.15	9.76
10	11.00	11.14	11.24	11.36	....	....	10.75	....	....	10.88	11.18	11.70	12.20
12	13.08	13.24	13.36	13.50	....	....	12.75	....	....	12.96	13.36	13.96	14.48
14	15.14	15.32	15.48	15.64	14.5	....	14.75	15.00	15.25	15.04	15.56	16.26	16.88
16	17.20	17.40	17.60	17.78	16.5	....	16.75	17.00	17.25	17.12	17.76	18.50	19.30
18	19.28	19.50	19.74	19.92	18.5	....	18.75	19.00	19.25	19.18	19.94	20.78	21.74
20	21.34	21.60	21.84	22.06	20.5	....	20.75	21.00	21.25	21.26	22.14	23.06	24.18
24	25.52	25.78	26.08	26.32	24.5	....	24.75	25.00	25.25	25.38	26.50	27.64	28.96
30	31.76	32.06	32.40	32.74	30.5	....	30.75	31.00	31.25	31.80	33.08	34.58	36.24
36	37.98	38.30	38.72	39.16	36.5	....	36.75	37.00	37.25	38.18	39.66	41.60	43.48
42	44.20	44.56	45.08	45.56	42.5	....	42.75	43.00	43.25	....	....	....	....
48	50.52	50.84	51.42	51.92	48.5	....	48.75	49.00	49.25	....	....	....	....

Permanent repairs can be made using the same size of pipe as the broken main, and this is the desirable objective. However, in some cases, the urgency of the repair may indicate the desirability of a semipermanent repair, using a smaller diameter main. If the inserted length is not too long, the repair can be made more quickly by using a reducer fitting and a smaller diameter of new pipe. In some cases, the loss of head will not seriously reduce the efficiency of the distribution system. A knowledge of the adequacy of the existing water mains is necessary, therefore, to determine what parts of the system can be repaired with smaller pipe.

The Committee has made a careful study of the procedure for repairing a broken water main, with the cooperation of the principal water pipe manufacturers and others. A so-called "adapter sleeve" is proposed, as shown in Fig. 2. This is similar to standard mechanical joints made by a number of water pipe manufacturers. However, the proposed adapter sleeve would be somewhat larger in diameter than the standard for a given size of pipe, and would have a longer straight section back of the joint so as to embrace the jagged ends of the shattered pipe. A somewhat larger rubber gasket will be required and the rubber used, perhaps, should be somewhat softer than standard. A few generally standard fittings will also be useful in the emergency repair of water mains, such as mechanical joint split sleeves, test plugs, test caps, and the like. The procedure described has been referred to manufacturers of cast-iron pipe who are making preparations to supply these fittings and to whom inquiry should be made. It is highly desirable that a sufficiently high priority be granted by the proper authorities.

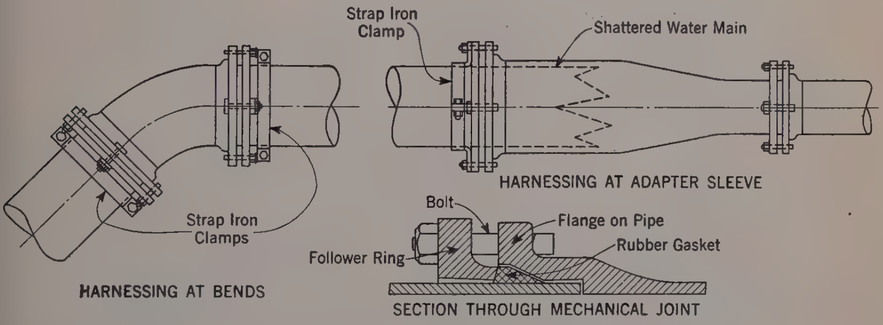

Fig. 2.—Adapter Sleeve for Repairing Water Mains

Suggestions as to the installation of the new section of main in a crater are shown in Fig. 3. The use of tie rods extending from one joint to the next is not indicated, but in certain cases where pressures are unusually high, or the subsoil is unusually yielding, the use of tie rods may be imperative.

The use of such a method of repair and of the suggested adapter sleeve will not only permit a much quicker repair, but will also reduce the number of fittings that must be kept on hand. An advantage of the reducer type of sleeve is that the number of different sizes of new pipe to be kept on hand will also be less.

In all cases, facilities should be provided for the heavy treatment of water in the pipe lines adjacent to where repairs have been made, with chlorine compounds. In England, it is considered desirable to make repairs permanent wherever possible.

18. Protective Coverings

Certain parts of sanitary engineering structures are located underground, such as valves, valve chambers, and wells. Tests of protective coverings under direct hits by bombs from airplanes have been made by the War Department at Edgemont, Md., and have been reviewed by the Committee. Through the courtesy of Maj. F. J. Wilson, M. Am. Soc. C. E., a suggested protective covering has been prepared, as shown in Fig. 4. In this illustration, the covering is for an underground valve chamber adjacent to a main pumping station. The length and width of the covering should be related to the vulnerability and importance of the structure. The cutoff trench along one side is indicated as a means of dampening the destructive vibrations that might come from a bomb falling outside of the protective covering. Such a protective covering for a well or an isolated valve would cover a smaller area.

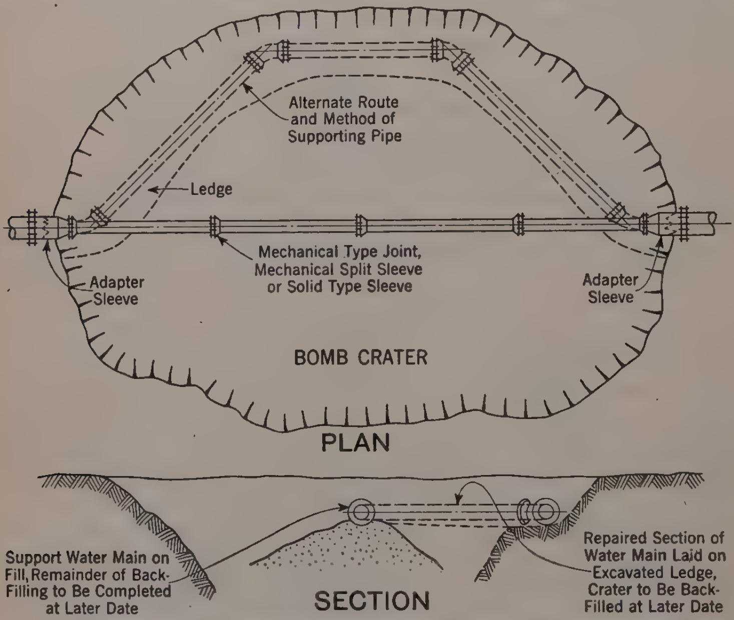

Fig. 3.—Method of Repairing Water Main Through Bomb Crater

Fig. 5 shows a typical simple bomb shelter, suitable for construction adjacent to a pumping station, sewage treatment plant, or other important structure. Another type of bomb shelter is shown in Bulletin No. 1, "Civilian Defense Protective Construction—Structure Series." The arrangement shown in Fig. 5 is taken from "Civil Air Defense," by Augustin M. Prentiss, Lieutenant-Colonel, General Staff Corps, U. S. Army. A simple

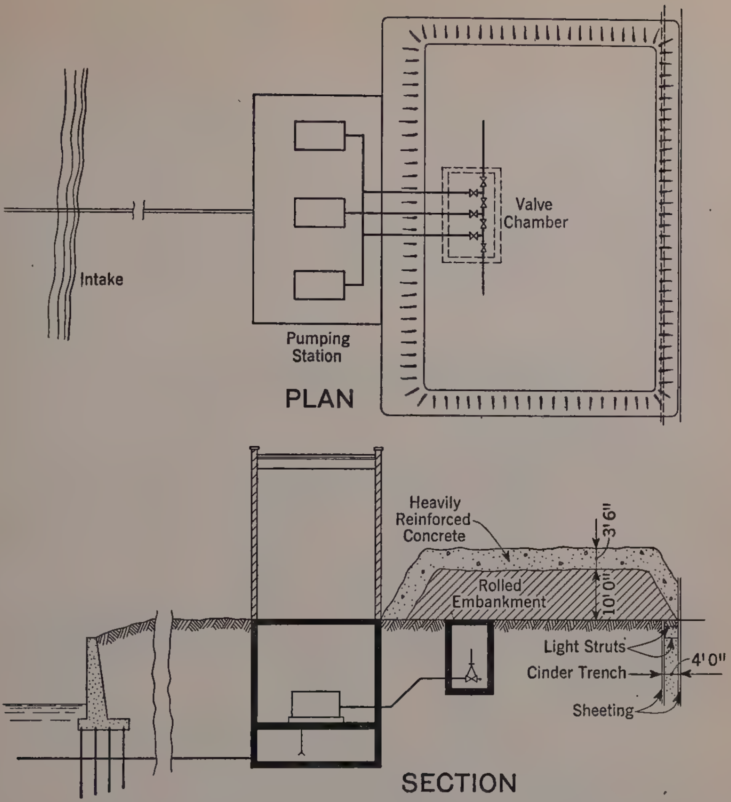

Fig. 4.—Protective Covering of Subsurface Structure

hand-force pump should stand in the trench. No ventilating equipment is included, as the shelter is not intended for long use. Figs. 4 and 5 are suggestive only and are not applicable to all parts of the United States and all kinds and locations of structures. In some cities and for some structures a heavier protective covering will be required, and in other places something lighter. Thus, on the Pacific Coast, the concrete course may be 4 or 5 ft thick, and in other less exposed localities only about 2 ft thick.

Splinter-proof walls around important pieces of equipment are a valuable protection against the effect of an explosion near the structure or in the structure at some distance from the piece of equipment. A well-built brick wall 13 in. thick will provide such protection against splinters from a bomb, broken bits of masonry and blast. In effect, splinter-proof walls form stalls or rooms about the equipment.

19. Laboratory Procedure

Laboratory procedure is an important factor in the maintenance of a safe water in areas subject to sabotage or under aerial attack. The usual routine may be insufficient for the detection of suddenly introduced poisonous substances and of attempts to cause epidemics of disease by pollution. Water is poisoned by the introduction of chemicals injurious to the health of the population and intended to be fatal. Pollution and contamination is the introduction of live organisms that will cause disease. In addition, the normal laboratory procedure may be disrupted by a direct hit or explosion and British experience has indicated the desirability of having mobile or duplicate laboratories available. In some cities, use can be made of existing laboratories within the city, or in near-by cities, provided arrangements have been made. It should be noted that many commercial laboratories are not ordinarily equipped to make tests for poisons and contamination. Administrators should be advised to make the necessary preparations for this kind of laboratory work before the emergency arrives.

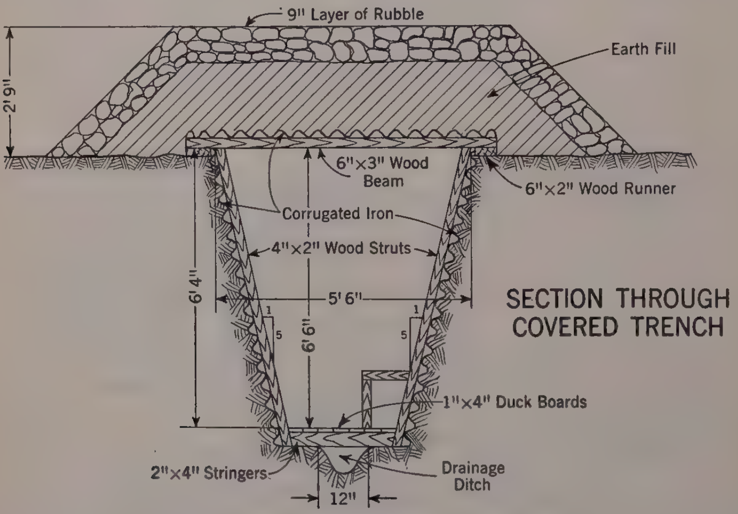

FIG. 5.—SMALL BOMB SHELTER FOR EMPLOYEES

If water is poisoned chemically, boiling cannot be counted on to remove the danger. In the emergency, gas may not be available. Therefore, it is necessary to know if the water is poisoned, and the population must be warned accordingly. Many of the normal water tests are of great value in detecting poisons and unusual contamination. During wartime, such tests should be made at more frequent intervals than otherwise. A brief summary of some of the more useful tests has been prepared for the Committee by R. F. Goudey, M. Am. Soc. C. E., as follows:

(*a*) *Chlorides.*—Low chlorides might indicate the addition of silver, mercury, or lead, whereas abnormally high chlorides might indicate the hydrolysis of war gases, most of which contain chlorine.

(*b*) *Sulfates.*—Low sulfates might be due to the addition of lead or barium salts, whereas an increase might be due to copper or zinc sulfate or the hydrolysis of war gases originally containing sulfur dioxide.

(*c*) *Oxygen Consumed* (Possibly Modified to a 5-Min Digestion Period).—Higher than normal values may be due to war gases, glucosids, alkaloids, and phenols.

(d) *Biochemical Oxygen Demand.*—Abnormally low results might be due to metallic poisons, glucosids, alkaloids, and war gases which have sterilized the test water. High results might be due to any organic poison in minor amounts where the procedure to test B.O.D. in the presence of free chlorine is followed. The delayed action of this test reduces its value in this case.

(e) *pH.*—A low pH-value of 4.0 to 7.0 might indicate hydrolysis of war gases, whereas an excessively high pH-value might be due to excess barium hydroxide.

(f) *Nitrates.*—High nitrates may be due to soluble forms of heavy metallic poisons.

(g) *Odor.*—Although odors from poisons may be cleverly masked, characteristic odors of various organic poisons can be easily identified. New tests for the detection of war gases absorbed into water should include a general test for heavy metals and a sensitive test for arsenic. Heavy metals are infinitely more toxic in organic combinations and can be added easily to water supplies in toxic quantities.

(h) *Heavy Metals and Arsenic by H_2S Precipitation.*—This test includes digestion of a 500-ml sample with about 3 ml of concentrated H_2SO_4 and sufficient $H\,N_3$ until the solution is colorless and SO_3 fumes have been produced, dilution of the residue with distilled water; and precipitation of metals with H_2S. The presence of heavy metals is indicated by variously colored precipitates. A whitish sulfur precipitate should be neglected; but if arsenic in more than 10 ppm is present, the sulfur precipitate will appear yellowish.

(i) *Arsenic (Sensitive Test).*—Take 100 ml of sample of any required concentration, and add 5 ml of concentrated H_2SO_4 and sufficient nitric acid and hydrogen peroxide to decolorize. Heat to SO_3 fumes in a Kjeldahl flask. Dilute with distilled water and, after adding 0.1 g of hydrazin sulfate, boil until SO_2 has been completely removed. Cool, neutralize with Na_2CO_3, add 30 ml of distilled water, 0.2 g of potassium bromide, and 5 ml of concentrated $H\,Cl$. Then titrate with 0.01 N $K\,Br\,O_3$ in the presence of methyl orange indicator. The titration is to the disappearance of color with continued additions of the indicator.

Already in some cities tests for poisons are made, as fear of sabotage has indicated the necessity.

A mobile laboratory unit may be required in the larger cities. One of the best has been developed and used by the U. S. Public Health Service in its pollution survey of the Ohio River.[1] The information regarding this has been made available to the Committee through the courtesy of Dr. Thomas Parran, Surgeon General, U. S. Public Health Service.

20. PROTECTION AND REPAIR OF DAMS

As many water supplies are derived wholly from storage behind impounded dams, and as many such dams are so located, that a major breach in them would cause a large loss of life and much destruction of property, their protection is often more important than the protection of the transmission lines leading from them. In some parts of the United States, an impounded supply, when lost, cannot be replaced until the dam has been repaired and the runoff of the stream has been allowed to accumulate behind the repaired dam. As the time for collecting runoff, generally, is during the late fall and spring, destruction of the dam and loss of stored water immediately following this period might involve a year or more of waiting to accumulate water, even if the dam is repaired promptly.

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[1] See *Journal*, A. W. W. A., April, 1941, pp. 638 and 643.

Insufficient experience is available to the Committee to permit a complete statement regarding this item. Further study and investigation are recommended. However, some suggestions have been made. Protective coverings will probably not be possible for the larger dams, although some dams might be given a substantial degree of protection along the lines indicated in Section 18.

Regarding the repair of dams, manifestly a large breach in a masonry dam would be difficult to repair while water is flowing through it. A small breach might be bridged by an emergency bulkhead, which could be kept floating in a vertical position back of the dam, ready for use. A comparatively small breach in an earth dam might be closed similarly by a floating bulkhead, which could be backed up promptly by a fill of sand bags. Earth embankments, however, are likely to ravel rapidly when exposed to a strong current and a repair of the foregoing type would have to be very prompt. Otherwise the breach would become so large that repair would have to be delayed until after the water had been largely drained from the reservoir.

Obviously, the protection and repair of important dams likely to cause loss of life and damage if broken require, in special degree, the general procedures outlined in foregoing sections, such as guarding against sabotage, arrangements for warning of the disaster, and readiness to supply water from an alternate source during the emergency.

21. MOBILE WATER TREATMENT UNITS

To meet certain emergencies caused by broken water mains and other disruptions of sanitary engineering services, mobile water treatment units are desirable. These are of two kinds. One comprises a portable chlorinating unit for sterilizing repaired water mains, suction wells, and other structures. The other is a mobile water filtration and sterilizing plant. Both types of unit are made in the United States.

It is understood that mobile water treatment units are part of the regular Army equipment, and thus may be considered by some as a military, rather than a civilian, item. However, aerial attack may come to cities not within Army combat zones, and it is desirable that such units be available either from military or civilian sources.

Portable chlorinating equipment used for sterilizing water mains and for other emergencies should include a portable gasoline pump, chlorinating equipment, chlorine, and appurtenances. In most cases, liquid chlorine should be used, but where it is not available, a hypochlorite solution may be useful.

A mobile water purification unit includes a gasoline-driven centrifugal pump, a rapid sand filter, chemical feed equipment, chlorinating equipment, and appurtenances. The capacity of such a unit depends chiefly on the amount and character of the turbidity of the raw water, but, under reasonably favorable circumstances, the quantity of treated water amounts to as much as 1,000 gal per 24 hr. This amount is very helpful in supplying emergency drinking water and hospital needs.

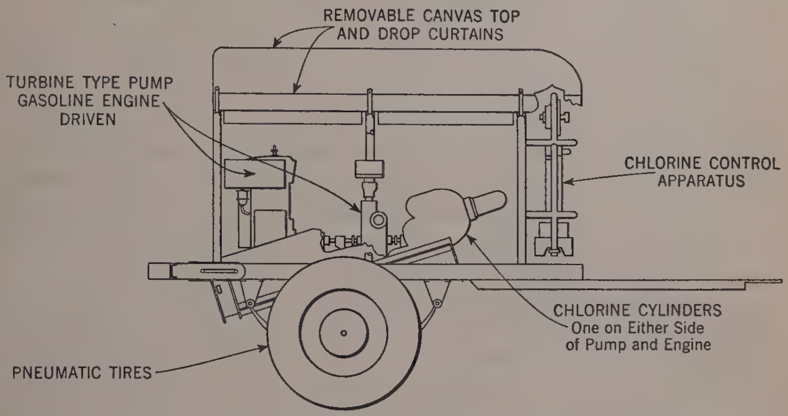
REMOVABLE CANVAS TOP AND DROP CURTAINS
TURBINE TYPE PUMP GASOLINE ENGINE DRIVEN
CHLORINE CONTROL APPARATUS
CHLORINE CYLINDERS One on Either Side of Pump and Engine
PNEUMATIC TIRES

FIG. 6.—PORTABLE CHLORINATING UNIT

A portable chlorinating unit is shown in Fig. 6 and a mobile water treatment unit in Fig. 7.

PART *B*.—SEWERAGE WORKS

Second in importance of the Sanitary and Public Health Engineering services are the sewerage works, comprising collecting, intercepting, and outfall sewers, sewage pumping stations, and treatment plants. The troubles that may result from bombing, attack, or sabotage and the protective and remedial measures are more or less similar to those described in Part *A*. Fig. 8 is an outline of a typical sewerage works.

22. COLLECTING SEWERS

In general, damage to or by collecting sewers is a result of explosion and the direct damage by poisoning is much less than in the case of water works. The results of the damage are the flooding of basements, the breaking of other underground conduits by washing out and undermining, damage to adjacent pavement, sidewalks, and foundations, and to the disruption of sanitary facilities in homes, hospitals, industries, and commercial establishments. Obviously, early steps to be taken include notification of the damage to all persons within the affected areas, and the temporary restoration of sanitary facilities. In some cases, temporary drainage can be provided by trenching from the upstream to the downstream side of the broken sewer. When large areas are affected, portable comfort stations should be made available and provision made for the use of sanitary facilities in adjacent districts.

Protective and remedial measures include, in the first place, a good map of the sewer system, showing the location, size, depth, and slope of the various sewers and appurtenances. The emergency supplies listed for the repair of distribution systems include most of the items needed for the repair of a broken sewer. However, the procedure differs in some respects and the stock of emergency equipment and supplies should include the following:

> (*a*) Concrete or steel pipe in several sizes and jointing materials. In repairing sewers, it is not necessary to fit the existing sewer exactly and, where necessary, two or more pipes of smaller diameter can be used to repair the broken section.
>
> (*b*) The stock of lumber should include a sufficient amount for the construction of a box conduit, should the diameter of the broken sewer be in excess of about 72 in. Such lumber could also be used for a temporary by-pass of the sewage during a permanent repair and for forms in replacing large concrete structures.
>
> (*c*) The pumping equipment should include well points and pumps with fittings suitable for attachment to well points. Well points are more likely to be required in the repair of sewers because they are often deeper than water mains.

One of the more serious annoyances resulting from a broken sewer is the flooding of basements. Portable pumps for dewatering flooded basements are a necessary part of the emergency equipment.

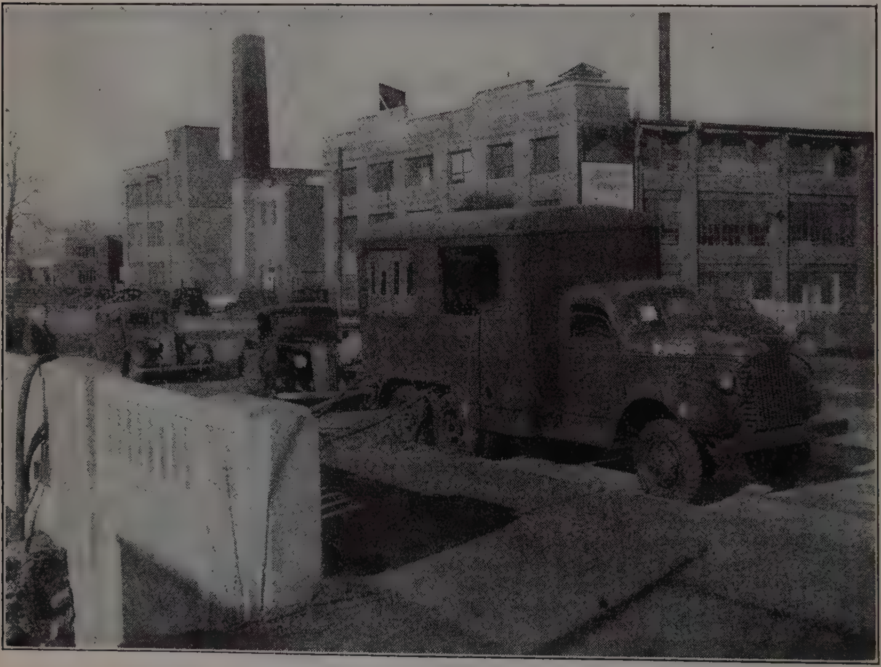

Fig. 7.—Mobile Water Treatment Unit

In some cities, it may be advisable to prepare plans for remedial measures in cooperation with local contractors, so that their equipment and skill may be available. These contractors should be instructed as to what may be expected of them in an emergency.

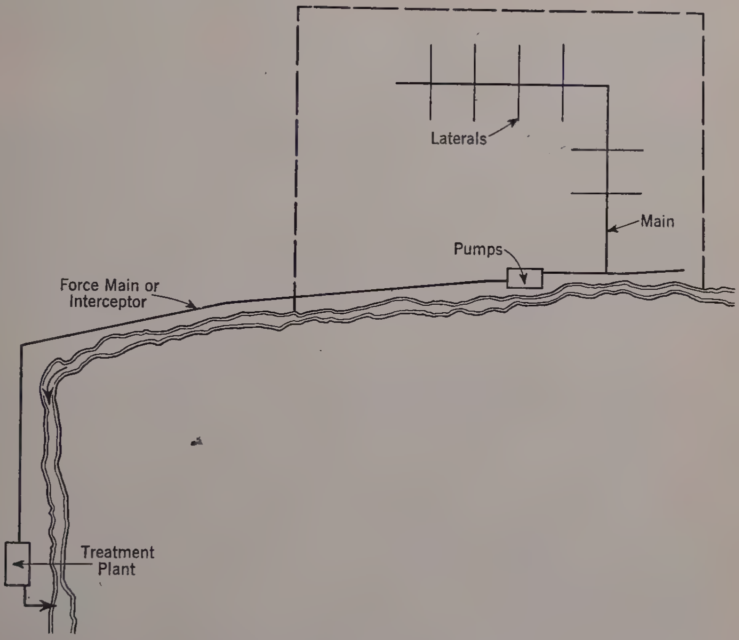

Fig. 8.—Typical Sewerage System

An important measure for the protection of collecting sewers is to provide substantial locks for manhole covers, especially those in isolated locations. There are a number of makes available.

A very good summary of experience in England with the repair of damaged sewers is abstracted from a recent issue of *The Surveyor*,[2] as follows:

> Repairs to sewers damaged by bombings are recommended by the Ministry of Home Security to be cared for by two parties—one party of six men including a foreman to rope off craters, clear debris, etc., and a larger party of ten or more workmen to make the repairs. Repair parties are considered important enough to defer the calling up for war service of the men who are engaged in this work.
>
> Adequate traveling facilities are believed to be essential.
>
> Allocation of personnel to districts is not recommended as the bombings are rarely distributed evenly. After bombings, the work of repair is distributed as fairly as possible with a view to avoiding unnecessary travel.
>
> General recommendations for inspecting the damage include:
>
> 1. An inspection by a technical assistant to determine and make record of the damage;

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[2] "Repair of War Damage to Sewers," by "Beta," *The Surveyor and Municipal and County Engineer*, London, July 11, 1941, p. 17.

2. Unless a bomb actually falls on the sewer it is unlikely that damage will be caused to a sewer located deeper than the bottom of the crater. If a bomb falls directly over the pipe line, the earth may be forced down, causing a fracture.
3. However, bombs splinter and sometimes penetrate downward 20 ft below the bottom of the crater, causing damage. Thus a complete inspection is necessary even in a shallow crater.
4. Generally, damage extends 10 to 20 ft beyond the crater.
5. Any manhole within 75 ft of a crater must be inspected.
6. Uncradled sewers usually suffer no more than those incased in concrete.
7. Damage may not appear promptly, thus requiring another inspection.

Temporary repairs are not recommended except (*a*) when a crater must be filled promptly for any reason, (*b*) when a large number of sewers are damaged at the same time and insufficient labor is available to repair all damage at the same time, or (*c*) when shortage of personnel and materials would delay permanent repairs for a long time.

Concrete cradles, steel, or spun-iron pipes, and, in each case, very careful backfilling are recommended for repairs. The most commonly used methods of dealing with the sewage flow are:

1. Shutting off the flow at the next upstream manhole and pumping out the sewer;
2. Exposing the ends of the fractured line and cutting a channel through the crater to carry the flow; and
3. Allowing the sewage to percolate into the soil.

The Ministry of Health requires certain information to be kept of the damage, as follows:

“1. Situation.
2. Date of damage.
3. Extent and nature of damage.
4. Cause of damage.
5. Original construction.
6. Construction after repair.
7. Date of commencement of repairs.
8. Date of completion of repairs.
9. Cost of repairs.
10. Foreman in charge.
11. Technical assistant in charge.”

Speed in making repairs is considered essential to the public health and morale in accordance with the policy of removing evidence of war damage as quickly as possible.

23. FLOODED BASEMENTS

Basements are flooded frequently when water mains and sewers are broken. After flooded basements are drained or pumped out, they should be washed, brushed, and scrubbed with water. Then a disinfecting solution of chloride of lime and water should be applied with a brush, broom, rag, or sprayer. An adequate solution can be made up with 1 lb of chloride of lime in about 8 gal of water. In regard to flooded furniture, clothing, and foodstuffs, the following precautions are suggested:

(*a*) *Furniture.*—

Curtains.—Boil all that can be boiled without injury to the fabric. Dry thoroughly in the open air and sunshine. Press with hot iron, or dry clean.

Rugs.—Flush with clear water while still on the floor. Dry thoroughly in the sunshine. Use a mild soap and lukewarm water to shampoo; then rinse and dry.

Furniture.—Wash with strong soap and water all surfaces that can be reached and will not be harmed, such as wood, metal, leather, cane, and composition materials. Wash upholstered materials and dry thoroughly, preferably in open air and sunshine.

(*b*) *Clothing.*—Boil immediately everything that can be boiled without injury. Otherwise, dry thoroughly, in the sunshine, all clothing that cannot be boiled. Then sterilize by pressing with a hot iron or by dry cleaning.

(*c*) *Foodstuffs.*—No foodstuffs subjected to contamination from sewage should be used unless such foodstuffs have been stored in watertight containers, in which case the outside should be thoroughly sterilized with boiling water or disinfectant. It is best to "play safe" and discard any questionable foodstuffs. No flooded foodstuffs should be sold to the public.

24. Intercepting and Outfall Sewers

The damage to intercepting and outfall sewers will be similar to that of collecting sewers; but it will affect wider areas. Therefore, the same measures for protection and repair should be applied. Intercepting and outfall sewers may be so located as to discharge, when broken, considerable quantities of raw sewage into a near-by water course. Plans should be made for disinfecting such discharges of raw sewage and of warning affected areas and services regarding the extent and duration of the pollution.

25. Sewage Pumping Stations

Sewage pumping stations are either main stations of considerable size, or small district pumping stations sometimes in isolated locations. In general, there will be a continuous personnel at a main pumping station, but district pumping stations may be visited only infrequently, sometimes once a day for oiling and inspection. At isolated pumping stations it would be advisable to provide steel shutters at windows and to replace wooden doors with steel doors. A high, substantial, protective fence should be provided. In addition, volunteer watchers in the vicinity of the district pumping station should be instructed regarding inspections of the pumping station property and notice to those responsible for its maintenance. In some cases, alarm systems should be provided to indicate when the pumping station goes out of service. In some cases, temporary outlets or by-passes can be provided.

Main sewage pumping stations are similar to water works pumping stations, and the same protective and remedial measures apply. However, in the case of a sewage pumping station, it may be feasible and necessary to by-pass raw sewage, and provision should be made for disinfecting this discharge.

26. SEWAGE TREATMENT PLANTS

Sewage treatment plants may be put entirely out of operation, or partly so, depending upon their size and the extent of aerial attack. Thus, adequate measures of by-passing not only the entire plant, but also parts of the plant, should be provided. Sewage treatment plants may be located so that camouflage may be effectively applied.

The use of by-passes requires an ample supply of chlorine so that by-passed raw or partly treated sewage can be thoroughly disinfected. Sometimes, parts of sewage treatment plants can be replaced quickly by using temporary earth lagoons, either for preliminary sedimentation, or for sludge digestion and storage. Preliminary plans and arrangements might well be prepared for such temporary lagoons. Interruptions in the power used for operating the treatment plant, especially if it is electric power, should be considered. For a sewage treatment plant having many points of use of electric power in small amounts, a stand-by generating unit is desirable.

Many modern sewage treatment plants require a constant supply of spare parts, lubricants, packing, and the like. During wartime, some of these supplies and repair parts will be difficult to secure, and careful watch should be kept on the stock room with reference to such supplies as the following:

Spare cutters, both fixed and moving, for comminutors and shredders
Spare links, flights, wearing shoes, and pins for sludge removal equipment
Shear pins
A set of brushes for fine screens
Nozzles for trickling filters
Diffuser plates and bolts for aeration tanks
Ball valves for sludge pumps
Fusible plugs for flame traps
For chlorinating equipment: Gaskets, packing, lead washers, pressure reducing inlet valves, bell jar, head tube, valves and flexible connections for feed lines, reducing valve union assembly
Valve channels and springs for vacuum pumps
Belts for Reeves drives
Cloths for vacuum filters
Packing, lubricants, laboratory equipment, etc.

Underground and separate storage should be provided for the more important supplies and repair parts. At sewage treatment plants, especially those remote from occupied areas, provision should be made for the temporary shelter of employees against bombs, as indicated in Fig. 5. In general, sewage treatment plants are similar to water treatment plants, and the same protective and remedial measures apply.

PART *C*.—REFUSE DISPOSAL

Refuse disposal is an important Sanitary and Public Health Engineering service, in that it affects many persons intimately. It may be disrupted by the destruction of the regular plants or places of disposal, or by the damaging of collection units at equipment yards or by the commandeering of mobile equipment for emergency uses. This service, however, is more likely to feel the effect of war by sudden shifts or enlargements in the population. Thus, the breaking of a water main and an extensive fire may force many people from their homes and cause a heavy concentration of garbage and other refuse along routes and in areas beyond their normal requirements. Protective and remedial measures are somewhat different from those relating to water and sewerage works.

27. Salvage

In modern total war, the conservation of materials for war equipment is of a great importance. British experience indicates that metals, waste paper, bones, and garbage are the most important refuse materials to be salvaged. The procedure, in general, is one of organization, and this should extend all over the United States. The first step is the separation by the householder. From then on, the method is one of routing the separate refuse materials from the house receptacle to the processing plant for ultimate use. This is a matter of organization. The British experience indicates that a moderate payment for salvaged refuse materials very much increases the quantity. Local committees to organize and invigorate the salvage and collection of metal, paper, bones, and garbage are helpful.

28. Feeding Garbage to Hogs

Disposal of garbage by feeding to hogs is a common experience in wartime. It is now the common method in England and was advocated during 1917 and 1918 by the Federal Food Administration in the United States. On December 7, 1917, this Administration held a conference in Chicago, Ill., of representatives of the government, sanitary engineers, and hog breeders and feeders, to determine the best method of feeding garbage to hogs. Obviously, the garbage should be collected reasonably free of other refuse materials and should be fed to the hogs in a reasonably fresh condition, not more than two or three days old, depending upon the season. A moot question is whether or not the garbage should be sterilized by boiling before it is fed to the hogs, and on this, opinion is divided. The conference held by the Food Administration in 1917 came to no definite conclusion.

It is said that 10% to 20% of the entire population of the United States is infected with pork worm, which is a parasite causing trichinosis. This disease would be eliminated if all types of pork meat were thoroughly cooked. Garbage-fed hogs contain the pork worm, but whether or not in greater proportion than corn-fed hogs is not definitely known. Boiling garbage before it is fed to hogs adds to the cost of disposal by this method and also makes it difficult for the hogs to select the better food from the garbage. Two measures for reducing the amount of trichinosis are the proper and complete cooking of pork meat, and the boiling or disinfecting of garbage before it is fed to hogs. Both measures are difficult to enforce completely. For instance, there are great numbers of farms and isolated residences where garbage is fed to hogs and where it would be very difficult to provide for boiling. Some people are also likely to undercook pork meat. No doubt, the exigencies of the wartime conditions will determine the question as to whether or not, and to what extent, garbage is boiled before it is fed to hogs. If a shortage of food is the important matter, then the effort to have the garbage sterilized will not be as strong as otherwise. If the healthiness of the people is much reduced and the amount of trichinosis increases, the measures for boiling the garbage will receive support. In any event, there is likely to be a considerable increase in the number of hogs fed with garbage, and state and local health officers should take steps to emphasize the necessity of using only well-cooked pork meat, if trichinosis is to be kept under control.

A method used in England for cooking and processing garbage for use as a food for hogs and poultry is described in *The Surveyor.*[3]

29. SANITARY FILLS

If garbage and other refuse are disposed of normally by incineration and the incineration plant should be put out of operation by explosion, arrangements should be made for disposal of the garbage by the method designated as "sanitary fill," which is, in effect, a regulated and controlled method of dumping. Land should be found where the garbage can be so disposed of during an emergency. Rules and regulations for the operation and maintenance of a sanitary land fill were stated by a Board appointed by Mr. Justice I. Wasservogel, of the Supreme Court of New York, as follows:

> "1. The disposal of wastes by the landfill method should be planned as an engineering project. Operation and maintenance should be under the direction of a sanitary engineer.
>
> "2. The face of the working fill should be kept as narrow as is consistent with proper operation of trucks and equipment in order that the area of waste material exposed during the operating day be minimal.
>
> "3. The exposed surface should be covered with earth as promptly as is consistent with proper operation and at the close of each day's operation both the surface and the face of the fill should be completely covered, the object being to make a closed cell of each day's deposit.
>
> "4. Sufficient standby equipment should be provided to prevent delay in covering, due to breakdowns or peak loads.
>
> "5. Waste building material, concrete or other bulky waste material which may furnish rat harborage should not be used for the final surface or side slopes, but should be promptly incorporated within the fill.
>
> "6. The final covering for surface and side slopes should be maintained at a depth of approximately twenty-four inches.
>
> "7. In case the finished fill has a boundary side slope, the toe of the slope should terminate in a sand and gravel-filled ditch. This will prevent raveling of the toe with exposure of some of the waste material, will prevent the burrowing of rodents and finally will obviate puddles by permitting seepage from the fill to be absorbed into the ground.
>
> "8. Spraying of the exposed waste material and adjacent surfaces should be used when necessary to allay dust.
>
> "9. As a rule, the layer of refuse should not exceed an average depth of about eight feet after compacting. Where deeper fills are necessary, the filling should be carried on in stages.

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[3] *The Surveyor and Municipal and County Engineer*, May 16, 1941, p. 323.

"10. Control over the blowing of papers should be adequately maintained by the use of movable snow fencing.

"11. While the maintenance of proper earth covering as hereinbefore recommended will to a large extent prevent fires, water under pressure should be available for fire fighting purposes. If scavengers are tolerated, they should be adequately supervised.

"12. All collections of surface water resulting from these landfill operations should be drained, filled or treated with effective chemicals so as to prevent mosquito production or allay disagreeable odors.

"13. Where necessary, effective steps should be taken to prevent floatage of waste material into open water.

"14. Inspection for and control of rodents should be maintained until the fills are stabilized.

"15. After the active period of filling operation is completed a maintenance program should be continued until the fill has become stabilized so as to insure prompt repair of cracks, depressions and erosion of the surface and side slopes.

"16. Studies of the varied problems involved in landfill operations should be continued and should include researches into the biological, chemical and physical activities, as well as the engineering, economic and administrative aspects of the subject."

The Committee agrees, in general, with the foregoing recommendations, but suggests that, for temporary emergency use, the final covering for the surface and the side slopes of the fill need not exceed about 12 in. Furthermore, if the fill is not located adjacent to houses, the spraying operation may be omitted. Except in large undertakings, the Committee suggests that the depth of refuse, especially of garbage, be not more than about 2 ft.

30. Collection Service

The most important part of refuse disposal is the collection of the materials from the dwellings and other points of production throughout the community. The usual precautions regarding the trustworthiness of the personnel should be applied. Of particular importance is the storage of the collection units at separate points, rather than in a common yard, during wartime emergencies. Provision should also be made for garage service at more than one location. If, under normal conditions, collection vehicles are serviced at a municipal garage, arrangements should be made for this service, if necessary, by private garages and repair shops located at some distance from the municipal plant. Methods of adapting the collection units to a salvaging program should be studied and provisions made for any alterations so required.

31. Disposal Works

In many cities, garbage and rubbish are disposed of together by incineration. Salvaging paper and feeding garbage to hogs may prevent the continued use of this method. Some garbage is disposed of by reduction for the recovery of grease and low-grade fertilizer. This method may be continued by the federal government during wartime. In general, one or two disposal works are sufficient, although in very large cities several are required, to a total of more than twenty in New York, N. Y. A direct hit during an aerial attack might destroy a refuse disposal plant completely, or some damage might result from a near-by explosion. Thus, the protection and remedial measures described for water and sewage pumping stations should be applied.

32. Street Cleaning

The cleaning of streets will not be much disturbed by attack during wartime, except as the physical efficiency of the personnel may decline. However, if and when there are sudden large shifts in population, the necessary amount of street cleaning may exceed the facilities of the normal personnel and equipment. It is not unlikely that populations of moderate sized cities or sections of larger cities may be increased two or three times for short periods. The official responsible for street cleaning in each community should consider such a situation and make definite plans for meeting it. The organization of volunteer workers is suggested. It will be necessary to have available sufficient street cleaning equipment, especially of the hand-use type, for the volunteer crews.

Also, the street cleaning organization may be required to clear streets of debris after air raids. Occasionally, this will require a very large amount of work. Overtime effort is indicated, together with volunteer workers. Such emergency conditions should be met with plans and procedures made in cooperation with other municipal services, so that all available men can be used efficiently.

PART *D.*—OTHER FACILITIES

33. Public Comfort Stations

Public comfort stations do not comprise an item of first importance in Sanitary and Public Health Engineering. Under normal conditions, they serve a minor part of the population and it would not be a serious inconvenience if they were out of commission for a short time. However, stations of this sort will be much needed during wartime emergencies.

Emergency Conditions and Needs.—During and shortly after an air raid, sections of a city may be without sanitation service. This will result in an additional load in adjacent areas. There may be occasions also when the population of adjacent damaged or ruined cities will migrate to adjoining communities thus severely overloading such areas. Furthermore, during the sudden establishment of war industries and establishments, there may be large temporary increases in the population. Conditions such as these place a heavy burden on the sanitary services rendered by public comfort stations. Very unsanitary conditions may result if precautionary and remedial measures are not taken promptly.

Plans and arrangements should be prepared for temporary public comfort stations, preferably with flush toilets connected to sewers, but otherwise with chemical closets or earth pits. These are designated as "sanitary privies" or latrines of the "can type." Locations for their temporary erection, and arrangements for the necessary materials, should be part of the general program. In general, the location of all available sanitary services should be known and directions to them given to the public during emergencies by guides and signs.

If the services rendered by public comfort stations should be seriously and extensively destroyed by aerial attack, undoubtedly the need for adequate street cleaning will be accentuated and this should be considered in the procedures to provide adequate emergency street cleaning.

A very good publication which will be useful to local committees in the construction of privies or latrines is to be found in *Supplement No. 108* to the U. S. Public Health Service Reports.

34. MILK SUPPLIES

Milk supplies are generally subject to public regulation and control. They comprise an important aspect of Sanitary and Public Health Engineering services, especially during short periods of emergency increases in population. They are also subject to poisoning and contamination by saboteurs. Therefore, any plans for civilian protection in wartime should include consideration of milk supplies.

35. *Production and Distribution.*—The production of milk is generally in scattered locations remote from centers of population. It is not highly vulnerable to aerial attack, therefore, but may be quite easy of access by saboteurs. The milk may be pasteurized and bottled near the point of production or near the point of distribution. The distribution plants are generally located in or near centers of population. Therefore, they are vulnerable to aerial attack and explosion and may be put out of service. In most large cities there are a number of such pasteurization, bottling, and distribution plants. Therefore, the disruption of one plant would place an additional load on the others. Sudden large increases in the population will place similar loads on the milk supply. It has been found in areas adjacent to defense projects that a rapid increase of the milk demand and a corresponding expansion of the milk supply facilities are likely to lower the quality of the milk, because the inspection and regulating service does not expand as rapidly as the demand and the resulting facilities. During periods of national defense, in advance of wartime conditions, officials responsible for the production of safe milk supplies should plan the necessary steps to meet sudden increases in the demand for milk and milk products, so as to avoid danger to the public health. In certain vulnerable regions, consideration should be given to policing the milk producing areas, first, perhaps, in cooperation with the state police and, later, by special guards.

36. ROOMING HOUSES AND RESTAURANTS

Rooming houses and restaurants are now generally subject to public regulation and control. They comprise an important aspect of Sanitary and Public Health Engineering services, especially under wartime conditions. Restaurants, in particular, may add considerably to the spread of influenza and other communicable diseases. In both cases, temporary overcrowding may require financial and planning assistance by the federal government. Both are subject to disturbance under attack and to poisoning and contamination by saboteurs.

Rooming Houses.—In most states, rooming houses, tourist accommodations, and overnight cabins for temporary use are subject to regulations by the State

Department of Health or other agency, and operate under a license. In some cases, the licenses are issued by the municipality, which may or may not be subject to the regulations of the state authorities. During wartime, the demand for such temporary housing facilities may become acute. Accordingly, it is recommended that local officials ascertain in advance the location and capacity of available private homes, assembly halls, gymnasiums, hotels, and the like which may be taken over for temporary housing. Personnel skilled in sanitary engineering and public health work should be made available to expedite emergency housing facilities and to inspect them for general cleanliness, airing, cleanliness of bedding, and the like. Spare furnishings, such as beds and bedding, will be required.

Restaurants.—Especially during emergency periods, restaurants may greatly increase the spread of influenza and other communicable diseases. Provision should be made to increase food inspection in proportion to the relatively large increase in food consumption likely to result from sudden large shifts in population. Many people may be deprived of their homes temporarily and so become dependent on restaurants for their food. Therefore, consideration should be given to the establishment of temporary boarding facilities. Emergency food handlers should also be thoroughly inspected to maintain a desirable standard of personal cleanliness and to avoid carriers.

Of particular importance is the thorough cleaning of utensils in restaurants. Reference should be made to the Acts of the Legislature of New York and other states and to their requirements for cleaning utensils in public eating places. Should influenza or any other communicable disease become epidemic, special instructions to housekeepers should be broadcast.

PART *E*.—SUMMARY

37. Funds and Budgets

It is apparent, from the foregoing sections, that emergency funds will be required to meet many needs—as, for instance, additional personnel in the sanitary services, training of that personnel, the purchase of necessary equipment and supplies, and emergency housing and food. Local committees should acquaint themselves with the methods to be followed and should prepare a local budget of the estimated amount of funds. Some or all of such emergency funds may be provided by the federal or state governments, but general plans for procedure should include a consideration of funds and budgets.

38. Suggested Procedure

To summarize the foregoing sections, the following procedure is suggested for municipalities and public service companies, in regard to protective and remedial measures for civilian protection in Sanitary and Public Health Engineering services. Perhaps at the start the importance of the community and its status as to vulnerability should be considered, so that the expenditures undertaken will be reasonable and proper. The principal steps in the procedure, more or less in the order in which they should be done, are outlined as follows:

(*a*) See that the record maps of the sanitary engineering services are complete and up to date. Arrange to have them kept from general distribution and have copies stored in safe places, so that if one set is lost, others will be available.

(*b*) Make a technical survey, to include some general plans, to determine the adequacy of the services to meet peacetime needs, with special reference to civilian protection in wartime. In some cases, immediate construction will be proper to increase the general efficiency and to facilitate its use in wartime. The survey and general plans should then be extended to those needed, should wartime needs prevail.

(*c*) Investigate the operating personnel for reliability and trustworthiness.

(*d*) Make a careful inventory of stocks on hand of supplies and repair parts; and acquire needed supplies.

(*e*) Make provision for alternate or secondary sources of water, at least as regards acquisition of land, rights of way, and permits.

(*f*) Set up laboratory procedures for the detection of poisons and sudden contaminations.

(*g*) Arrange for guarding and the peacetime protection of important structures, chiefly against sabotage.

(*h*) Make routine reviews of technical journals and keep a file of the latest developments in such important items as nature of attack, protective lighting, blackouts, etc.

(*i*) Establish working arrangements with adjacent or near-by communities where cooperation will be helpful in the maintenance of sanitary engineering services.

(*j*) Investigate existing power supplies and arrangements and make plans for alternate and emergency supplies.

(*k*) Prepare a general plan of procedure for local organization, including volunteer guards, workers, and messengers, emergency repair crews, and the use of contractors' equipment and personnel.

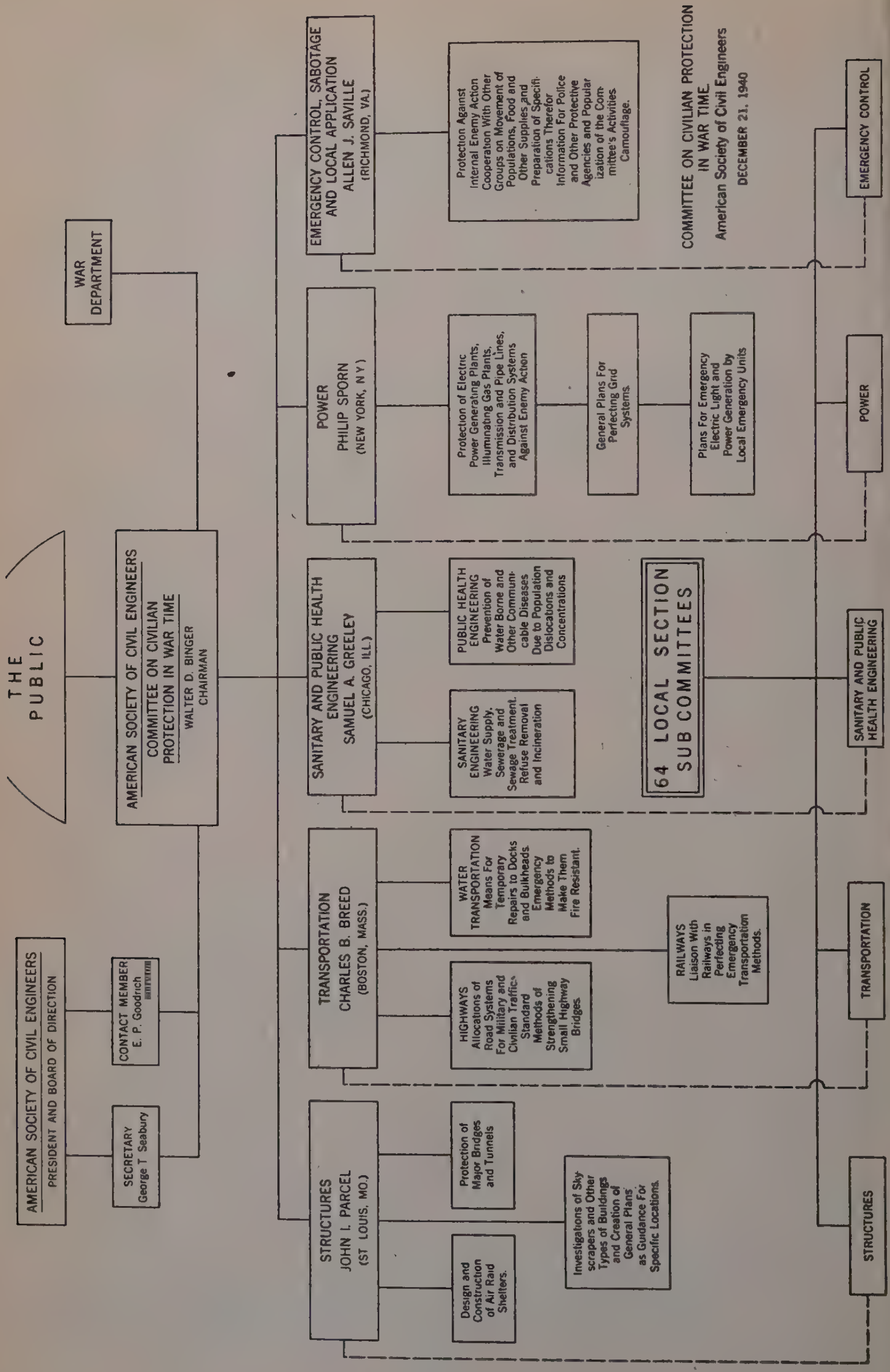
(*l*) Ascertain the names of representatives and how to effect cooperation with agencies such as local military establishments, the Federal Bureau of Investigation (FBI), state officials and departments, and local departments and organizations. These would include the city and state departments of health and the city fire and police departments.

(*m*) Provide gas masks, first aid supplies, and disinfectants at important structures.

(*n*) Provide an adequate supply of chlorine and other chemicals used in its treatment of water or for purposes of disinfection, or ascertain where this may be secured.

(*o*) Set up a committee to prepare a budget of emergency expenditures and a plan for providing the necessary funds.

(*p*) Appoint some one person to do the necessary work to accomplish the foregoing plan of procedure, and in all cases work through the local and state authorities.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American Society of Civil Engineers
DECEMBER 21, 1940
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ALLEN J. SAVILLE
(RICHMOND, VA.)
Protection Against Internal Enemy Action Cooperation With Other Groups on Movement of Populations, Food and Other Supplies and Preparation of Specifications Therefor Information For Police and Other Protective Agencies and Popularization of the Committee's Activities. Camouflage.
STRUCTURES
TRANSPORTATION
SANITARY AND PUBLIC HEALTH ENGINEERING
POWER
EMERGENCY CONTROL

FIG. 9

39. ORGANIZATION OF NATIONAL COMMITTEE

The foregoing Progress Report has been prepared as part of the work of the National Committee of the Society on Civilian Protection in War Time. At the meeting of the Society in New York City in January, 1941, this Committee was constituted, with Walter D. Binger, M. Am. Soc. C. E., as Chairman, and Ernest P. Goodrich, M. Am. Soc. C. E., as contact member.

When the Committee was organized (see Fig. 9), its potential influence was projected over the entire Nation through five national Committeemen:

Chairmen	Divisions
John I. Parcel	Structures
Charles B. Breed	Transportation
Samuel A. Greeley	Sanitary and Public Health Engineering
Allen J. Saville	Emergency Control and Local Application
Philip Sporn	Power

and the subcommittees of 64 Local Sections of the Society.

This organization was perfected under the close consultation of the Secretary of War, who appointed the Committee Chairman, Mr. Binger, as the Chairman of a National Technological Civil Protection Committee, to be constituted as follows:

Walter D. Binger, American Society of Civil Engineers, Chairman
W. H. Carrier, American Society of Heating and Ventilating Engineers
Harry E. Jordan, American Water Works Association
A. B. Ray, American Institute of Chemical Engineers
Abel Wolman, American Public Health Association
James L. Walsh, American Society of Mechanical Engineers
W. Cullen Morris, American Gas Association
Frederick G. Frost, American Institute of Architects
Scott Turner, American Institute of Mining and Metallurgical Engineers
E. M. Hastings, American Railway Engineering Association
John C. Parker, American Institute of Electrical Engineers
The contact member, U. S. Department of War

Descriptive articles on the activities of this organization have appeared in *Civil Engineering* for January (page 64), February (page 125), April (page 251), and June (page 378), 1941.

The work of the National Committee has been assisted by members of the 64 Local Sections of the Society, some of whom have contributed much to the Progress Report.

Respectfully submitted,
SAMUEL A. GREELEY, *Division Chairman,*
Sanitary and Public Health Engineering
Division of the National Committee of the
American Society of Civil Engineers on
Civilian Protection in War Time.

January 15, 1942.

APPENDIX

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(22) **"Conservancy Sanitation—Adaptation to Modern Requirements: Problems Connected with Air Raid Precautions,"** by Henry H. Clay, July 7, 1939, p. 25.
(23) **"Repair of War Damage to Sewers,"** July 11, 1941.
(24) **"A Method for Cooking and Processing Garbage Used as Food for Hogs and Poultry Adopted in England,"** May 16, 1941, p. 323.
(25) **"Sewage Disposal and Drainage in War Time—Treatment Difficulties—Design and Location of New Works,"** November 28, 1941, p. 181.
(26) **"Salvage as It Should be Carried Out by Cleansing Departments in War Time,"** by Colin H. Macfarlane, Director of Public Cleansing, May 31, 1940.
(27) **"Air Raid Shelter Sanitation,"** April 12, 1940.
(28) **"Barnsley Market Place Air Raid Shelters: Sanitary Accommodation,"** June 14, 1940.
(29) **"Municipal Salvage,"** February 9, 1940.
(30) **"The Special Importance of War Time Municipal Salvage—Helping Industry and the National Economy,"** by A. L. T., November 3, 1939, pp. 379–381.
(31) **"The Gravesend Shelter,"** Bolted Arch Construction—Flooding of Garden Shelters—The Annex Shelter—The Hove Indoor Shelter—Garden Shelters, January 31, 1941.
(32) **"Kitchen Waste as Pig Food—Dudley's Scheme,"** February 7, 1941.
(33) **"A Year of Salvage Activities,"** by W. E. Cook, Supt. of Public Cleansing, Bingham Urban Dist. Council, February 7, 1941.
(34) **"A Surveyor's View of Air Raid Precautions,"** February 14, 1941.

(35) **"Fire Protection of Structural Steelwork,"** February 28, 1941.
(36) **"Preferential Refuse Collection,"** February 28, 1941.

N.—Water Works Engineering

(1) **"Bombs Explode in England but Water Service Continues,"** March 12, 1941, pp. 289, 310.
(2) **"Emergency Disinfection of Water,"** December 18, 1940, pp. 1574, 1599.
(3) **"Guarding Water Systems Against Sabotage,"** October 23, 1940, pp. 1354–1356; November 6, 1940, pp. 1414, 1417, 1418.
(4) **"Interconnections of Municipal Water Supply Systems,"** January 15, 1941, pp. 72–74.
(5) **"Water Supply in Time of War,"** by F. F. Longley, October 23, 1940, pp. 1344–1348, and 1367–1368; discussions: November 6, 1940, pp. 1405–1406.
(6) **"Notifying Public of a Major Break in the System,"** November 20, 1940, pp. 1464–1465.
(7) **"War Time Protection of Water Works,"** by Joseph D. Lewin, May 7, 1941, p. 508.
(8) **"When Bombs Fell on Barcelona,"** by R. Perera, March 12, 1941, pp. 282–286.
(9) **"Protection of Water Supply Stressed at New York Meeting,"** January 1, 1941, pp. 28, 32, 37.
(10) **"Steps to Safeguard Plants Described by FBI Agents,"** by J. W. Randlette, January 1, 1941, p. 27.
(11) **"Safeguarding Water Systems Against Possible Sabotage,"** by F. J. Reny, November 20, 1940, p. 1480.
(12) **"How Rochester Disinfected Main Accidentally Polluted,"** January 15, 1941, pp. 83–84.
(13) **"War to Speed Consolidation of English Water Systems,"** December 3, 1941, Vol. 94, No. 25, p. 1497.
(14) **"Should War Come,"** by Victor J. Wilmoth, December 18, 1940, pp. 1570–1574.

O.—General

(1) **Publications of Massachusetts Committee on Public Safety,** The State House, Boston, Mass.
(2) **Report of Meeting of New England Water Works Association,** June 1, 1940.
(3) **"Losses of Life in Modern Wars"** (Austria-Hungary-France), by Gaston Bodart, Carnegie Endowment for International Peace.
(4) **"Epidemics Resulting from Wars,"** by Friederich Prinzing, Carnegie Endowment for International Peace.
(5) **"Civil Defense,"** compiled from articles written by Joseph A. Baer, U. S. A. Chief of Staff, 2d Corps Area (Civil Defense Office Headquarters, Second Corps Area—Governors Island, N. Y.).

(6) **"Losses of Life Caused by War,"** by Samuel Dumas and K. O. Vedel-Petersen, Chapter II—Carnegie Endowment for International Peace.

(7) **"National Defense in the New York Region in Relation to City and Regional Planning,"** *Bulletin No. 54*—Regional Plan Assn., New York, N. Y.

(8) **"Utilities and the War,"** by Herbert Corey, *Public Utilities Fortnight*, February 13, 1941.

(9) **"A Major Disaster Emergency Plan,"** and **"The Work of the Fire Service During and Following Air Raids,"** International Assn. of Fire Chfs., 24 West 40th St., New York, N. Y.

(10) **"Civil-Military Relations,"** prepared for Committee on Public Administration of Social Science Research Council, Public Administration Service, Chicago, Ill.

(11) **"Protection of Water Supplies in War Time,"** by E. Sherman Chase, *Journal*, Maine Water Utilities Assn., Augusta, Me., April, 1941.

(12) **"Cities and the National Defense Program,"** *Report No. 146*, May, 1941, The Am. Municipal Assn., Chicago, Ill.

(13) **"Massachusetts State Emergency Plan,"** Mass. Dept. of Public Health.

(14) **"British Cities at War,"** report by James L. Sundquist, Am. Municipal Assn., 1941, Chicago, Ill.

(15) **"New Jersey Official Plan of Defense—New Jersey Defense Council,"** Trenton, N. J., August, 1940, February, 1941.

(16) **"Trichinosis, A Preventable Disease,"** by Donald Augustine, *The Commonhealth* (Mass.), Vol. 26, No. 1, 1939.

(17) **"Studies on Trichinosis,"** by Willard H. Wright, *Reprint No. 2172*, Public Health Repts., Vol. 55, No. 24, June 14, 1940.

(18) **"The Emergency Repair of Cast Iron Mains,"** by W. Boden (Tech. Engr., Stanton Iron Works), *A Handbook of Pipes and Joints.*

(19) **"Toxic Contaminants of Drinking Water,"** by L. T. Fairhall, Prin. Industrial Toxicologist, Div. of Industrial Hygiene, Nat. Inst. of Health, U. S. Public Health Service.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

DRAINAGE OF LEVEED AREAS IN MOUNTAINOUS VALLEYS

Discussion

BY MERRILL BERNARD, M. AM. SOC. C. E.

MERRILL BERNARD,[10] M. AM. SOC. C. E.[10a]—Although Mr. Williams has confined his discussion to projects created by the construction of levee systems of considerable magnitude and importance, similar situations of lesser extent are to be found in nearly all of the great metropolitan areas and in many smaller cities and towns. His paper is a contribution to the solution of these problems and should be an encouragement to those who would treat them rationally rather than empirically.

In planning for the disposal of tributary flow within leveed areas, the ideal situation to be met is that of a synchronization of flow which, by virtue of a consistently shorter concentration period, would insure the discharge of the lateral flow under positive head through automatic floodgates for all conditions of outside stage. These circumstances may be encountered on small watersheds as headwaters are approached, but are never sufficiently realized to preclude the necessity of resorting to other means of control discussed by the author. In addition to the six methods mentioned by him, opportunity might occasionally present itself to consider the use of a secondary levee provided with gates and pumps. Under reasonably favorable channel and surface slope conditions, liberal gate dimensions to insure maximum duration of flow under natural head, and minimum supplemental pumping, the effect would be to enhance the utilization of areas dedicated to temporary storage and to reduce the depth and duration of flooding on developed, contiguous, low-lying areas. The storage basin itself could be closed in without reducing its capacity by substituting depth created by the secondary levee for area covered by the unconfined basin. The sequence of operation would be: (1) Gravity flow through upper and lower gates, (2) lower gates closed with the reversal of head, (3) lower pumping started, (4) upper gates closed with reversal of head, (5) upper pumping started, (6) upper pumping stopped, and (7) lower pumping stopped.

NOTE.—This paper by Gordon R. Williams, Assoc. M. Am. Soc. C. E., appears on pp. 3–16 of this issue of *Proceedings*.

[10] Superv. Hydrologist, U. S. Weather Bureau, Washington, D. C.

[10a] Received by the Secretary November 13, 1941.

The effect, on cost, of dividing the pumping installation into two units, the additional cost of constructing the secondary levee, and the greater operating cost, if any, would have to be weighed against the benefits of added protection in each particular case.

The author has capitalized reasonably on the fact that flooding in eastern Pennsylvania is confined to a rather definite "flood season." The 12-hr precipitation record at Harrisburg, Pa., demonstrates the differences between annual rainfall and that within the adopted flood season:

F	All storms or maximum yearly	All storms or maximum seasonal
2	2.0	1.5
5	2.7	1.8
10	3.5	2.0
40	5.0	2.4

The column headings indicate a confirmation of the author's opinion that the selection of the maximum yearly or seasonal values, when treated through Eq. 1, will produce the same frequency-depth relations as if all rainfall amounts within the record were considered, any differences being confined to frequencies of less than 2 years. Also, the differences in the values of the second and third columns represent reduction in design values gained by restricting the frequency series to the flood season. This in effect is offsetting the higher intensities of summer-type rainfall with the greater reductions for infiltration prevailing during the off-flood season. The frequency of the greater amounts occasioned by the inclusion of all storms within the record is reduced by increasing the value of m in the frequency equation.

The value of 1 in. in 24 hr for the contribution of melting snow to runoff seems somewhat low. There is little secure knowledge of this phenomenon but, on watersheds sufficiently small to allow snow-melt runoff from the entire area within a melting period, greater amounts could be expected. The average temperature for the locality that can occur with snow cover and coincidental rainfall could be as high as 55° F for a 24-hr period. The general opinion is that from 0.05 to 0.15 in. of snow-melt runoff per degree-day can be expected, depending upon the state of the snow mantle. Under these conditions runoff depth from 1.1 to 3.5 in. is possible. This does not include the contribution of free water present in a mantle of snow of low quality.

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DISCUSSIONS

RELIABILITY OF STATION-YEAR RAINFALL-FREQUENCY DETERMINATIONS

Discussion

BY KATHARINE CLARKE-HAFSTAD

KATHARINE CLARKE-HAFSTAD[44] (by letter).[44a]—The discussions have demonstrated a recognition on the part of engineers and hydrologists of a need for more reliable estimates of the intensities and frequencies of rainstorms through adequate analyses of existing rainfall data. Although this closing discussion must deal chiefly with points in the paper on which the writer and the discussers disagree, the writer wishes to express here her appreciation of the interest in the paper and of the constructive comments that will lead to a better understanding of the station-year method.

Because of the nature of the publication and the interests of the readers, it was felt that the mathematics and statistics should be simplified and confined to a minimum. As Mr. Smith states, "* * * the paper has its background in the mathematics of statistics, the major part of which the author has intentionally omitted * * *." Therefore, the statistical part of the paper is unsatisfactory to the statisticians.

At the present stage of development of statistics there is no method for the rigorous determination of the amount of dependence in data that are nonrandom in both time and space, such as station-year data. Nevertheless, the widespread use of the station-year method by hydrologists necessitated some means of obtaining a measure of persistence in such data and consequently of the reliability of the frequencies determined by the method, and led to the adaptation of the technique described in this paper.

Some of the questions raised in the discussions, such as the one pertaining to the relation between "analysis of variance," discussed by Mr. Thom, and Bartels' method of measuring dependence, had been carefully studied by the

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NOTE.—This paper by Katharine Clarke-Hafstad was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by Paul V. Hodges, M. Am. Soc. C. E.; February, 1941, by Messrs. C. S. Jarvis, and Howard W. Brod; March, 1941, by Messrs. Merrill Bernard, and Charles F. Ruff; April, 1941, by Eugene L. Grant, M. Am. Soc. C. E.; May, 1941, by Messrs. Waldo E. Smith, and Robert L. Lowry, Jr.; and June, 1941, by Herbert C. S. Thom, Esq.

[44] Asst. Climatologist, Section of Climatic and Physiographic Research, Div. of Research, SCS, Washington, D. C.

[44a] Received by the Secretary December 9, 1941.

writer but were considered inappropriate for discussion in such a paper. They are covered more specifically in a paper[20] by L. R. Hafstad.

The writer protests the statement by Mr. Thom that "Statistical analysis * * * must be applied only to such problems for which it can give solutions—problems involving random variables." Although the philosophies and methods of modern statistics were developed on the premise that only random variables are involved, completely random or independent data are seldom found in natural phenomena. The outlook for progress in climatic and hydrologic research by the use of statistical analysis would be dark indeed if this tool could not be used on observations possessing persistence. It seems to the writer that it is imperative, first, to determine by statistical methods whether or not the data are non-random in character, and then either to eliminate any persistence before using the statistics applicable only to random data, or to devise new techniques for dealing with non-random observations.

Evidently Mr. Thom misunderstood the manner in which the data in Table 1 were derived. Every occurrence of 1 in. or more of rainfall in a day at every station, regardless of the amounts the other stations recorded, was tabulated. If these data were plotted on a correlation table such as Fig. 9, not all the points would fall in square A. Thus the data which the writer used are not comparable to those of Mr. Thom and the comparison he made of the square correlation ratio and Bartels' statistic is irrelevant.

Mr. Thom's criticism, that much of the dependence is ignored in taking only daily amounts of 1 in. or more, is not clear, since the purpose was to determine the amount of persistence only for those storm days on which 1 in. or more fell at one or more stations. If a lower limit, say 0.80 in., had been selected, one would, of course, expect to find more persistence in the data than in the case of the 1-in. amounts.

Throughout the paper the writer was careful not to use the word "storm" with reference to the daily occurrences of rain, because storm data are not available; and if they were, the storms would be of various lengths and statistical analysis very much complicated by this fact. Mr. Thom states "that only a single rainfall value at a particular station should be taken from each storm." Probably Mr. Thom means that only one daily amount should be used from each storm. The assumption has been made in the paper that the daily interval constitutes the single event, and it is known that these events are not mutually exclusive, or independent. The result of this temporal dependence is to make the data, such as those in Table 1, less random than if there were areal persistence only. Although the effects of these two types of persistence cannot be separated by the method proposed, at least the final measure of dependence, N_d, upon which the reliability of the frequency depends, does include all the persistence in both time and space which tends to decrease the reliability of the frequency.

The writer stated (see heading: "Factors Affecting the Accuracy of Frequency Determinations: Number and Distribution of Stations") that two general patterns of rainfall distribution result from two general types of physical processes taking place in the atmosphere—convection and frontal activity. Mr. Thom assumes that frontal activity consists only in the "simple process of lifting up a frontal surface," and on the basis of this assumption criticizes the writer for stating that frontal action is one of the chief causes of intense precipitation. Mr. Thom would attribute most high intensity rainfall to horizontal convergence.

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[20] "On the Bartels' Technique for Time-Series Analysis, and Its Relation to the Analysis of Variance," by L. R. Hafstad, *Journal*, Am. Statistical Assn., Vol. 35, 1940, pp. 347–361.

The writer disagrees with Mr. Thom's concept of "frontal activity." Frontal action should include all processes that may take place in the atmosphere in and near frontal surfaces—vertical motion, convergence, and convection. This concept is supported by Sverre Petterssen, who states that:[45]

> "This kind of precipitation [intermittent or continuous from a continuous cloud cover of the alto-stratus or nimbo-stratus type] is caused by the slow upward movement of a large mass of air, due to convergence in the horizontal motion of the air. * * * this type of precipitation is called frontal precipitation."

Messrs. Brunt and Douglas state,[39] "It is now widely recognized that continuous rain is usually associated with 'fronts,' or lines of separation of air masses of different temperature."

Convergence is a part of frontal activity. It is, also, an important factor in releasing and maintaining convection. The intensity of rain at frontal surfaces does not depend entirely on the amount of convergence and the upslope motion. "It depends also on the slope of the frontal surface, the moisture content, and the stability of the air."[46] Although high intensity rains have occasionally been observed to result from horizontal convergence alone with no clearly defined fronts in the vicinity, by far the greater number of occurrences of high intensity rains are associated with the passages of fronts or with convective activity.

In his discussion, Mr. Hodges has attempted to make a practical application of errors in pluvial indexes to the problem of flood forecasting. Unfortunately, the writer cannot agree to the transformation from error in frequency to error in pluvial index suggested by Mr. Hodges. The standard errors given in Fig. 2 are simply errors in the spacing in time of any given amount of rain, based on the assumption that the number of occurrences of that amount in the station-year record is known, and that the frequency distribution of these amounts follows the Poisson law. Since the analyst does not know the error in the number of occurrences of the given amount from which number the pluvial index is derived, nor the form of the distribution of rainfall amounts against number of occurrences, he is not justified in assuming that he knows anything about the error in the pluvial index.

The writer had never heard of the method for deriving rainfall frequencies from a station-year record mentioned by Mr. Jarvis—that of assuming that the second highest value in a station-year record has a frequency of once in one half the total years of record, the third highest a frequency of once in one fourth the total years, etc. Indeed, there is no logical reason for determining an average by such a method. If the procedure suggested by Mr. Jarvis were extended, the fifteenth highest value in a 500-station-year record would have an average frequency of once in 11 days. The usual method would give an average frequency of 33.3 years. The difference in these two values could scarcely be considered as the "range in uncertainty regarding the probable error."

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[45] "Introduction to Meteorology," by Sverre Petterssen, McGraw-Hill Book Co., Inc., New York, N. Y., 1941, p. 38.

[39] *Memoirs*, Royal Meteorological Soc., Vol. III, No. 22, p. 34 *et seq.*

[46] "Weather Analysis and Forecasting," by Sverre Petterssen, McGraw-Hill Book Co., Inc., New York, N. Y., 1940, p. 425.

Mr. Brod questions the application of station-year pluvial indexes to large drainage basins. As was stated in the paper (see heading "Factors Affecting the Accuracy of Frequency Determinations: Number and Distribution of Stations"): "no 2° quadrangle of the earth's surface will have uniform characteristics of rainfall"; and, as Mr. Bernard emphasizes, the station-year method should be applied only to a region having in all its parts the same rainfall regime. Although no detailed studies have been made to determine the size of such areas, it is certain from meteorological considerations that they are of extremely limited extent, certainly not as large as the area included within a 2° quadrangle of latitude and longitude. Professor Grant's study[12] concerned only the determination of areas homogeneous with respect to the frequency of very high rates of rainfall. The writer may perhaps be pardoned for emphasizing once more that areal dependence in the station-year data is just as important a factor detracting from the value of the method as is the dependence between successive years and successive storms and the dissimilarity of rainfall stations mentioned by Mr. Brod.

It may be of interest to engineers to know that the Climatic and Physiographic Division of the Soil Conservation Service is conducting an investigation of rainfall probabilities. At present (1942) the project is confined to the development of an improved statistical technique for determining the probabilities of various rainfall intensities for selected stations in the United States. The data being used are the actual maximum amounts for selected time intervals, read from the triple-register sheets of the recording rain gages.

Mr. Bernard has contributed a very clear explanation of the manner in which a station-year record is synthesized. One reading it cannot fail to understand why and how there is introduced into the composite record a certain amount of "dependence" between the station records. From Mr. Bernard's theoretical considerations of the morphology of rainstorms it is evident that the hydrologist needs to know much more about the size of areas receiving various depths of precipitation in rainstorms, and about the relations between the intensities and total amounts that actually occur in the storms and those that are recorded by various spacings of rain gages. Until some studies of these features are made one cannot hope to obtain reliable values of average frequencies of various amounts of rain, or of total actual precipitation falling on drainage basins.

Mr. Ruff's question as to whether, from a measure of the dependence between stations, one could estimate the frequency and amount of rain at surrounding stations when a given station has its 50-yr rainfall, must be answered in the negative. If the form of the mean depth-area distribution curve for rainstorms were known, one could calculate the probability that an area of a certain size received the 50-yr amount when it was observed at a single station; but as Mr. Bernard points out in his conclusion, the position of the station receiving the 50-yr amount with respect to the pattern of the rainfall for this single storm will not be known from the observations at this one station, and therefore its observed depth will give no clue to the depth or frequency of the rain occurring at surrounding stations.

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[12] Discussion by Eugene L. Grant of "Rainfall Intensities and Frequencies," by A. J. Schafmayer, M. Am. Soc. C. E., and the late B. E. Grant, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 384.

Mr. Smith has compared the standard errors for the mean areal extent of rainstorms in South Carolina and Iowa (see Fig. 6 and supporting text) and concludes "that the standard error as found for 4 in. for South Carolina is definitely out of line with the others * * *." One is not justified, however, in comparing in this manner the standard errors in average extent of rainstorms for the two areas, since the length of record and the spacing of stations is not the same for the Iowa quadrangle and for the Carolina quadrangle. The Iowa rainfall data are for twenty-one stations for 32 years, or 672 station years, whereas the Carolina data are for fifteen stations for 20 years, or 300 station years. Since the size of the standard error of the mean number of stations receiving a given amount of rain in a day is a function solely of the number of observations of that amount in the total record, it is not unreasonable to find that the error for the 4-in. amounts in the Carolina quadrangle is larger than for the Iowa quadrangle which has the longer record.

Mr. Smith has indicated, also, that there is some difficulty in distinguishing between the definitions for N_a and N_d used in the paper. Symbol N_a is the notation for the average number of stations receiving a given amount of precipitation in a calendar day, and N_d is an index of the number of statistically dependent ordinates. In the theoretical case described under the heading "Determination of Dependence Between Stations," the writer set up an artificial situation with a known amount of dependence between stations by assuming that a certain amount of rain always fell at exactly the same number of stations (5) on each day of rain, and that the days on which this amount occurred were distributed at random among the years of record. Some misunderstanding of the definitions may have resulted because in this theoretical case $N_a = N_d$. In the application of the test for dependence to actual rainfall data, such as Table 1, there will be dependence both between years and between stations and $N_a \neq N_d$, as was stated under the heading "Determination of Dependence Between Stations."

Mr. Smith is correct in stating that several rain gages measured the rainfall associated with the flood on the Republican River in May, 1935. However, it is doubtful whether this flood would have been expected from the measurements of these gages alone. Although unofficial measurements in stock tanks, cans, etc., in the area of highest intensity reached 13 in. (one even 24 in.), the highest amount recorded by a Weather Bureau gage was 5.05 in.

It is unfortunate that the writer defined the pluvial index (see "Introduction") as "the maximum rainfall to be expected with a certain frequency." As Mr. Lowry states, the correct definition as used in the Miami study is "that amount which may be expected to occur or to be exceeded * * * on an average of once in the frequency cited." In the case cited by Mr. Lowry for Quadrangle I-14 of the Miami Charts, 12.6 in. is the pluvial index corresponding to an average frequency of 100 years. However, the "most probable value" of a distribution is usually defined as the mode, and if this definition is used it is not certain that the most probable maximum value to be expected in a time interval of 100 years will be higher than the pluvial index of 12.6 in. Mr. Lowry probably referred to the mean maximum value, which will be somewhat larger.

Although very little is known concerning the distribution of rainfall amounts, there is evidence to support the assumption that they form some type of exponential distribution. In the following example (contributed by George Blumenstock, Jr., of the Climatic and Physiographic Division of the Soil Conservation Service), a simple exponential distribution

$$f(x) = e^{-x} \qquad \text{(9)}$$

is used to clarify the distinction between the value of the variable x (the pluvial index) corresponding to a mean recurrence interval, T, and the most probable maximum and mean maximum values of x in an equivalent interval.

First, determine the value, K, of the variable, x, for which the mean recurrence interval is T (that is, the value of x which may be expected to be equaled or exceeded on the average once in every T trials). This value, K, is defined by the relationship:

$$\int_0^\infty e^{-x}\,dx = T\int_K^\infty e^{-x}\,dx \qquad \text{(10)}$$

from which:

$$K = \log_e T \qquad \text{(11)}$$

In a random sample of T events drawn from the population $f(x)$, the probability distribution of the largest value will be given by the relationship:

$$\phi(x) = T\,F^{T-1}(x)\,f(x) \qquad \text{(12)}$$

The mode, u, of this distribution is the most probable maximum value, and will be given by the solution of the equation:

$$\frac{T-1}{F(x)}f(x) + \frac{f'(x)}{f(x)} = 0 \qquad \text{(13)}$$

This leads to the result:

$$u = \log_e T \qquad \text{(14)}$$

It is to be seen, then, that at least for the type distribution $f(x) = e^{-x}$ for any given recurrence interval, T, the corresponding value, K, is exactly equal to the mode, or most probable value, of the probability distribution of the largest value of x for an equivalent time interval.

In the case of rainfall data, however, the mean of distribution (Eq. 12) will often be a more significant measure of the maximum value of x to be expected in T trials. R. A. Fisher and L. H. C. Tippett[47] have shown that for an exponential distribution similar to Eq. 9, the mean, $\bar{u}$, of Eq. 12 for large values of T is given by:

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[47] "Limiting Forms of the Frequency Distribution of the Smallest and the Largest Member of a Sample," by R. A. Fisher and L. H. C. Tippett, *Proceedings*, Cambridge Philosophical Soc., Vol. 24 (1928), pp. 180–190.

$$\bar{u} = u + \frac{c}{\alpha} \qquad (15)$$

in which u is the mode; c is Euler's constant, 0.57721+; and α is defined by:

$$\alpha = \frac{f(u)}{1 - F(u)} \qquad (16)$$

For the distribution defined by Eq. 9, it follows from Eq. 16 that α is equal to 1, and, since $K = u$:

$$\bar{u} = K + c \qquad (17)$$

The distinction between the value K corresponding to a mean recurrence interval, the most probable maximum u, and the mean maximum $\bar{u}$ is now clear. For the original distribution $f(x) = e^{-x}$ one may now say:

(1) The value K corresponding to a mean recurrence interval, T, is (Eq. 11): $K = \log_e T$. This is also the most probable maximum value, u.

(2) The mean maximum value $\bar{u}$ in T trials is

$$\bar{u} = \log_e T + c \qquad (18)$$

For example, where T is assumed equal to 100 (a mean frequency of 1 in 100) the value of K and u is $\log_e 100$, or 4.605, but the mean maximum, $\bar{u}$, in 100 trials is 5.182.

It should be emphasized that these relationships have been selected only to show that a distinction between the three values does exist. In the case of rainfall data the exact character and extent of the distinction depend upon the distribution $f(x)$ which may be selected as representative of the total population of rainfall amounts from which the sample is drawn.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

FORT PECK SLIDE

Discussion

By T. A. Middlebrooks, Assoc. M. Am. Soc. C. E.

T. A. Middlebrooks,[24] Assoc. M. Am. Soc. C. E. (by letter).[24a]—Mr. Feld declares that "large variation in natural soil cushion always causes trouble." The writer cannot agree with this statement. If trouble was to be experienced from a wide variation in the thickness of the natural overburden, the slide should have occurred on the left abutment instead of the right abutment. Between stations 12+00 and 19+00, the thickness varied from zero to about 25 ft, a change of approximately 3.5 ft in 100 ft; whereas, between stations 77 and 84, the thickness varied from 100 ft to 10 ft, a change of approximately 13.0 ft in 100 ft. It is evident, therefore, that this change had nothing to do with the failure. The writer has investigated numerous earth slides and has found none that could be remotely attributed to a change in thickness of the overburden in the manner described by Mr. Feld.

Mr. Feld's conclusion that the fill was too rigid to take up the differential settlement can scarcely be substantiated. Actually, differential settlement between stations 10+00 and 20+00 was quite small—approximately 0.25 ft (3 in.) in 100 ft. Such a small differential movement could not cause shear failure in either the sand shells or the core.

He concludes correctly that the surcharge load increased the hydrostatic pressure in the shale and caused uplift (excess hydrostatic) pressures in the base. The fact that the surcharge load, as pointed out by the writer, was the sole cause of the excess hydrostatic pressure in the shale has been definitely proved by further investigation since the slide.

The writer is at a loss to understand how Mr. Feld would increase the stability of the structure by the use of "Vertical cleavage joints, with proper shear dowels," and the "construction of the dam between stations 20 and 85 first." It is customary practice to place cleavage joints and shear dowels in concrete structures. However, their use in the Fort Peck Dam would have

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Note.—This paper by T. A. Middlebrooks, Assoc. M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Jacob Feld, M. Am. Soc. C. E.; April, 1941, by Joel D. Justin, M. Am. Soc. C. E.; May, 1941, by Messrs. William Gerig, Alfred J. Ryan, and Glennon Gilboy; September, 1941, by Frank E. Fahlquist, Assoc. M. Am. Soc. C. E.; and December, 1941, by Irving B. Crosby, Affiliate Am. Soc. C. E.

[24] Prin. Engr., Corps of Engrs., War Dept., Soil Mechanics Section, Washington, D. C.

[24a] Received by the Secretary November 5, 1941.

served no useful purpose and would have been entirely impracticable to construct. Observations on the excess hydrostatic pressure in the shale, since the slide, have shown conclusively that any reasonable extension of the construction period would not have materially increased the strength of the shale and bentonite, or reduced the excess hydrostatic pressure.

In Mr. Feld's closing paragraph he has evidently confused stress with strength in his reference to "'erection' stresses." The stress in a given earth structure will be practically the same directly upon completion as it will after the shrinkage in the dam and settlement in the foundation have been taken up. The greatest variable is the strength of materials involved, which reaches a maximum only when the materials are 100% consolidated.

The writer was pleased to have Mr. Justin's discussion and his agreement as to the primary cause of the slide. As Mr. Justin states, considerable confidence can be placed in the redesigned section since the slide was used as a full-scale test for the determination of the over-all strength of the materials involved.

Mr. Gerig's statements concerning the stability of dredged sand, and his conclusion that the sand shell did not flow or liquefy, should be noted carefully. Since Mr. Gerig's experience checks with the conclusions drawn from tests conducted on the shell material and observations of the undisturbed cores, it can be concluded, without question, that the sand shell did not flow or liquefy.

The writer agrees fully with Mr. Gerig's conclusion that "entire dependence cannot always be given to the results of laboratory tests." However, laboratory tests are essential where there is no past experience upon which to rely, and they are a necessity to the practical engineer in all other cases to supplement his judgment and experience.

Mr. Ryan's clear description of the action of excess hydrostatic pressure in the voids of a soil is most interesting. The writer agrees that for most cases the static slide method will give satisfactory results provided the distribution and amount of excess hydrostatic pressure and the reduction in strength due to progressive movement are known. An investigation reported in 1941 by the U. S. Waterways Experiment Station (Engineer Department)[25] shows that, with the proper determination of the excess hydrostatic pressure and remolding effect due to progressive movement during construction, the circular slide method will give reasonable results.

Evidently Mr. Ryan found in his stability analysis (Fig. 15) the same thing as the author—namely, that by proper manipulation of the shearing strength of the core he could get the two methods to check. It is well to emphasize that for a correct analysis by any method the distribution and amount of excess hydrostatic pressure and the decrease in strength due to progressive movement must be known. In the redesign of the section these factors were taken into consideration by using over-all strength values determined from the actual slide. The writer regrets that there are no additional data on the excess hydrostatic pressure that would be helpful in a stability analysis over that given in the paper.

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[25] "Investigation of the Pendleton Levee Failure," report by the U. S. Waterways Experiment Station, Vicksburg, Miss., 1941.

Mr. Gilboy has given a clear résumé of the action of the slide, and it is noted that he is in general agreement with the writer in all main features. He disagreed, however, on the action of the shell and transition after the initial failure had occurred. The action of this material after the initial failure is of minor importance in analyzing the cause of the Fort Peck slide; however, it is of major importance in the design of future hydraulic-fill dams in earthquake regions. Mr. Gilboy concludes definitely that there was liquefaction of the transition zone material and possibly the shell material during the slide.

The density of fill does coincide closely with the "minimum critical density" as pointed out by Mr. Gilboy. The margin of safety, however, is much greater than this comparison would indicate, since the critical density varied with every change in the material in the hydraulic fill. It is not an exaggeration to state that the critical density actually varied from the minimum to the maximum within a few inches of fill. Liquefaction over a sufficiently large area to cause trouble would have to include the strong with the weak. It is obvious, therefore, that the average critical density should be compared with the average fill density.

There are several other pertinent factors to this question, which were not mentioned by Mr. Gilboy, that should be emphasized. Undisturbed (frozen) 36-in. cores taken from the shell and from transition material after the slide showed no evidence of liquefaction, which is the major one of these factors. The fact that the "quick" condition on the surface, noted by Mr. Gilboy, was in the area where the transition zone and shell material sloughed into the settling core pool, as the shell moved out in a body, is another important factor. The core pool water trapped in the fissures and crack of the material was forced up through the mass, and a "quick" condition resulted at the surface. The remaining shell material, even that resting on the transition zone, moved out as a solid mass with very little disturbance on the surface except for tension cracks. In view of the overwhelming evidence that shows that the dredged sand did not liquefy and the obvious lack of any positive evidence that it did, it must be concluded that there was no liquefaction of the dredged sand.

The writer agrees fully with Mr. Gilboy that the water in the core pool settling into the crack left by the outward movement of the shell probably gave a little extra push that would not have been present in the rolled fill. However, the writer is confident that there would have been no basic difference in the nature of the slide except that the maximum distance that it moved might have been somewhat less (probably 900 ft instead of 1,000 ft).

Landslide activity stressed so strongly by Mr. Fahlquist had no bearing on the true cause of the slide. However, since this condition has been brought into the discussion, with the inference that it was not investigated, some comments are essential. Old landslides and faulting were quite extensive in the Bearpaw shale, and it was evident that a thorough investigation was absolutely necessary. Therefore, one of the first steps in the investigation of the foundation condition was to outline in detail all the faults and landslides. These conditions in the slide area were mapped very much in detail prior to the slide by explorations, including a tunnel into the abutment, shafts, and trenches, as well as core boring, and by actual excavation of the shafts, tunnels, and portal. It was concluded from the voluminous data available prior to the slide that the faulting and old landslide activity in this area would not cause any trouble, and the detail investigation after the slide substantiated this earlier conclusion. Even though there was considerable faulting in this area, as shown in the full report[12] on the slide, the Report of the Board of Consulting Engineers did not mention it as a possible cause of the slide. They did consider the possibility of grouting the so-called "A" fault on the left abutment, but after a thorough study had been made no grouting was considered necessary.

The 30-ft thick zone of blocky weathered shale found on the abutment, and referred to by Mr. Fahlquist, played only a secondary part in the slide. Movement definitely started around Station 15+00, due to the weak zone in the foundation, and as the mass moved out it carried a large part of this weathered shale with it. The writer is confident that the weathered shale in the abutment had no influence on the initial movement of the slide.

Mr. Fahlquist leaves the impression that the slide was due to some mystic geological condition involving old landslide activity that cores and undisturbed samples of the shale would not disclose, but that geological study of the region would. If there is any one point that stands out clearly from the writer's experience at Fort Peck, it is that a general geological study of the region is worthless for the investigation of any definite area for probable slides, unless it is accompanied by undisturbed samples of the rock in questionable areas. In any such investigation, the strength of the rock is essential, and it cannot be determined by a geological study of the region or even the immediate vicinity.

If, in all cases, the entire questionable mass was removed, as recommended by Mr. Fahlquist, there would, of course, be no need for undisturbed samples of the rock. However, at Fort Peck this procedure definitely was not economically feasible, and it bordered on the impossible.

In his closing statement, Mr. Fahlquist makes the following recommendations:

> "When such conditions are recognized, the only adequate exploration to confirm or disprove preliminary geologic conceptions is twofold: (1) A few well-selected and carefully executed borings to outline the extent of such geologic conditions, and (2) real life-size excavations to explore the condition fully, in area and in depth."

These recommendations are inadequate since they do not provide a means of determining whether the rock is strong enough to be stable. The Fort Peck slide is ample testimony to this fact, since the only thing missing from the original geological investigation was sufficiently large, undisturbed samples to determine the true character and strength of the upper 10 ft of the rock. This is the outstanding lesson to be learned by geologists and engineers from the Fort Peck slide.

Mr. Crosby minimizes the importance of the excess hydrostatic pressure in causing the slide, because other natural slides in this area occurred without this condition existing. The writer had no intention of leaving the impression that all slides were caused by the presence of excess hydrostatic pressure.

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[12] See "Report on the Slide of a Portion of the Upstream Face of the Fort Peck Dam," Corps of Engrs., U. S. Army, July, 1939.

Old slides along the river were definitely caused by undercutting which is an entirely different situation from that which existed in the Fort Peck slide. The slide at Snake Butte, Mont., was an old one that started moving again when the upper part was heavily loaded with quarry waste. These slides present no evidence from which one could logically conclude that excess hydrostatic pressure in the Fort Peck slide was not important.

It is agreed that the water that was present to develop the excess hydrostatic pressure did not come from the pores of the firm or sub-firm shale and bentonite seams. It came from the fractured and weathered zones in the shale and bentonite.

The writer agrees with Mr. Crosby in his conclusion "that 'liquefaction' of the bentonite previous to the slide did not exist." However, after a slight movement had occurred, the bentonite and shale were undoubtedly mixed with water and their strength considerably reduced. This, the writer believes, was the prime reason for the distance that the slide moved.

Mr. Crosby erroneously states that:

> "Numerous undisturbed samples of this shale and bentonite were taken, however, and numerous tests were made on them prior to the slide; but the true condition of the shale was not understood and the slide was not foreseen."

There were no undisturbed samples taken of the rock in the slide area, since the small core boring showed only sub-firm shale in this area. Sub-firm shale was found in numerous excavations to be a structurally strong rock. The geological explanation for the lack of weathered shale over the sub-firm shale in this area was that the river had scoured it off leaving only sub-firm shale. This conclusion at the time (without the benefit of hindsight) appeared entirely reasonable. Having been directly connected with the project for four years prior to the slide, the writer is firmly convinced that the actual strength and condition of the rock in the slide area could have been determined only by large undisturbed samples. It is rather far fetched to assume, regardless of how carefully the geology of the region was studied, that the true character and strength of the rock at any given location could be determined by this procedure.

The writer fully agrees that "Had the true conditions been understood, the dam could have been designed to meet those conditions safely." This is merely another way, however, of saying that hindsight is better than foresight.

It has not been the intent of the writer in this discussion to minimize the importance of thorough geological investigations. However, it is certainly false security to rely too much on a geological study of the region as proposed by Mr. Fahlquist and Mr. Crosby. In the future, under such circumstances, the writer will take large undisturbed samples of all weak rocks regardless of what the geologist might find in a study of the region.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

SOME ECONOMICS OF AIRPORTS

Discussion

BY W. WATTERS PAGON, M. AM. SOC. C. E.

W. WATTERS PAGON,[12] M. AM. SOC. C. E. (by letter).[12a]—Mr. Rudolph stresses the relation of ground time to air time for short hops between near-by cities. Mr. Baker deals mathematically with the same subject, by the reconstruction of his former figure to bring it into harmony with modern air speeds. The writer suggests that such discussions should emphasize that it is the difference in ground time to the air terminal as against the rail terminal, rather than the actual time. For example, the writer can reach the Baltimore (Md.) Municipal Airport in 15 to 18 min from his office, as opposed to 10 min or more to either of the rail terminals, and this ratio applies to almost all of the Baltimore business district. For the short hop from Baltimore to Philadelphia, Pa. (about 96 rail miles), the ground time in the two cities is 50 to 55 min to the air terminals as opposed to 25 to 30 min to the rail terminals; and the traveling times are slightly more than 0.5 hr and 1.5 hr, respectively, with an approximate saving of half an hour.

The City of Los Angeles, Calif., has studied the problem of providing air taxi service from one or more central airports to scattered "satellite" airports of small size. Small airplanes or autogyros will undoubtedly be used in the future to minimize the ground time to remote airports.

Mr. Rudolph's comments on the relative comfort of the airplane in mountainous regions are worthy of inclusion in the prediction of the growth of air travel. The writer is indebted to both discussers for their comments.

When a growth prediction has been made, from time to time it is desirable to check the actual progress against the prediction, to detect any changes of trend. This is especially true in the present matter because of the erratic growth curve from which the writer's curve has been projected. Table 4 brings the trends up to July, 1941, and for convenience the results for seven months in 1941 have been amplified in the same ratio that prevailed in 1940 to obtain

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NOTE.—This paper by W. Watters Pagon, M. Am. Soc. C. E., was published in February, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by Messrs. William E. Rudolph, and Donald M. Baker.

[12] Cons. Engr., Baltimore, Md.

[12a] Received by the Secretary December 5, 1941.

the figures for 1941. The number of passengers is slightly in excess of the prediction; the pounds-of-express curve has been much accelerated for eighteen months by war conditions. The data are taken from the "Survey of Current Business."[13]

TABLE 4.—COMPARISON OF ACTUAL WITH PREDICTED GROWTHS

Year	PASSENGERS CARRIED		PASSENGER MILES	EXPRESS (POUNDS)	
	Actual	Predicted	Actual	Actual	Predicted
1938	1,343,457	....	557,721	7,335,967	....
1939	1,876,051	2,130,000	749,785	9,514,300	6,390,000
1940	2,959,480	2,860,000	1,117,447	12,506,576	8,580,000
1941[a]	4,020,000	3,960,000	1,450,000	19,200,000	11,880,000

[a] Data complete to include July, 1941; the total for 1941 is estimated on that basis.

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[13] "Survey of Current Business," Bureau of Foreign and Domestic Commerce, U. S. Dept. of Commerce, the following Annual Review Nos.: February, 1939, Vol. 19, No. 2, p. 78; February, 1940, Vol. 20, No. 2, p. 77; February, 1941, Vol. 21, No. 2, p. 84; and October, 1941, Vol. 21, No. 10, p. S-21.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

TUNNEL CONSTRUCTION, SIXTH AVENUE SUBWAY, NEW YORK, N. Y.

Discussion

BY JACOB FELD, M. AM. SOC. C. E.

JACOB FELD,[7] M. AM. SOC. C. E.[7a]—At the time of organizing the construction procedure and methods for the tunnel sections of the contract for the construction of Section 10 of the Sixth Avenue Subway, a number of questions arose as to the most advisable procedure and methods, expected progress, as well as expected costs of each proposed method. Because of insufficient literature on the subject in attempting to determine what had been done under similar circumstances and what had resulted from the methods used, the writer decided that after this construction job was finished he would publish a paper, and list his own experience and the answers to the questions that arose. This paper is the result of that decision.

It might be worth while to enumerate some of the problems that arose, and their solutions:

1. Proper protection of soft and seamy rock at the tunnel portals, keeping in mind the close proximity of elevated column footings, street sewers, and utility lines, was provided by the construction of a reinforced concrete combination retaining wall and girder spanning across the tunnels. This method was very satisfactory and eliminated the expected trouble from the raveling of the rock at the portals.

2. The great variety of rocks encountered required a flexible drilling diagram, as well as flexible bracing methods. Especially helpful was the detail decided upon for connecting the individual units of the bracing frames.

3. The use of a full face drilling and shooting procedure—an unusual method in constructing tunnels within city limits—was found to be advisable and economical, and in no way increased the hazards of the work.

4. A flexible shooting cycle was necessary to coordinate the amounts of

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NOTE.—This paper by Jacob Feld, M. Am. Soc. C. E., was published in April, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1941, by John H. Myers, M. Am. Soc. C. E.

[7] Cons. Engr., New York, N. Y.

[7a] Received by the Secretary December 15, 1941.

dynamite necessary at each face with the restrictions placed upon blasting within the city limits.

5. The method of mucking required a study of different strengths of dynamite for the type of rock encountered, to avoid the necessity for breaking up rock too large for the mucking machine.

6. The lack of large areas, or easy access to the tunnel work, required an unusual disposal method; and the double-deck dumping board was found satisfactory and safe if only one dumping mechanism was provided.

7. The elimination of hauling the drill steel from the shaft for sharpening purposes was definite when traffic conditions on the street were considered and the decision to place the drill-sharpening plant beneath the decking at subgrade level proved not only economical but also safe. The predicted complaints from neighboring properties because of this operation did not materialize.

8. The special control of air and dust, resulting from the drilling operation, required considerable emergency research to avoid delay in the construction schedule. The method used has become standard and was accepted by the New York State Industrial Commissioner as an alternate to the only approved method at the time the rules were issued. Incidentally, the rules were issued after this contract was started.

9. The provision of small diameter shafts for inserting cable feed pipes to connect the street surface with the manholes adjacent to the tunnels by core drilling from the street surface was a substantial economy over the normal shaft-sinking method previously used.

10. The use of a concrete pump for placing all of the concrete within the tunnels, with the addition of an air booster to fill the irregular rock roof completely, was considered a novelty and was instituted in spite of the objections of the field organization. However, this method proved satisfactory and resulted in the use of the same equipment in many later jobs.

Many of these methods are new if compared with the reports of the rock-tunnel operations in the subway sections referred to by Mr. Myers in his discussion, even though the design of the subway tunnels has not changed.

This report proves that, contrary to usual feeling in the construction industry, satisfactory and uniform progress can be obtained in a short job by proper planning and, at the same time, can result in a low accident record. The accident frequency of one lost time accident per 25,000 man-hr is far below any record on similar work in previous years.

It is hoped that the data in this paper will be useful to engineers finding themselves in a similar position of planning and coordinating tunnel work within congested areas.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

OPERATION EXPERIENCES, TYGART RESERVOIR

Discussion

BY EDGAR E. FOSTER, ASSOC. M. AM. SOC. C. E.

EDGAR E. FOSTER,[6] ASSOC. M. AM. SOC. C. E.[6a]—The authors are to be commended for presenting a study of the results of the operation of one of the recently constructed multiple-purpose flood-control projects that the federal government has undertaken in the past decade. Few such projects have been completed long enough to observe the results of operation for a significant period of time. In the pioneer flood-control works constructed by the Miami (Miami Valley, Ohio) Conservancy District,[7] each dam was a single-purpose project only, since it used uncontrolled outlets for emptying the reservoir, and hence its operation could not constitute a complete model for the multiple-purpose projects.

The purpose of the Tygart Reservoir as stated by the authors is twofold—flood control and water conservation. The latter purpose provides water obtained from floods or high discharge in spring for the improvement of navigation, incidental benefits to water supply, and reduction of pollution during late summer. The utilization of the conservation water requires only releasing during periods of low stages at the controlling points along the stream below. The operation for flood control, on the other hand, must be planned carefully to obtain the desired result or even to prevent a worse flood. The seasonal basis for operating the Tygart Reservoir is depicted in Fig. 5. The plan of operation for both flood control and utilization of the conservation water has been skilfully developed.

Table 6 summarizes the reduction in flood peaks obtained at the site, at Lock and Dam 5, and at Pittsburgh, Pa., by operation of Tygart Reservoir. Generally, the reductions obtained at Tygart are much greater than at either Lock 5 or Pittsburgh, especially in the larger floods. Flood crests from storms

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NOTE.—This paper by Robert M. Morris, Esq., and Thomas L. Reilly, Esq., was published in April, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1941, by Nicholls W. Bowden, M. Am. Soc. C. E.

[6] Head, Flood Control Section, U. S. Engr. Office, Omaha, Nebr.

[6a] Received by the Secretary November 28, 1941.

[7] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1503.

centering over the headwaters of the Monongahela River, of course, will be reduced more effectively than storms centering elsewhere on the watershed. Although the reductions at Pittsburgh appear relatively small, they do not seem out of proportion to the drainage area controlled by the Tygart Reservoir as compared with the area above Pittsburgh.

TABLE 6.—SUMMARY OF FLOOD PEAK REDUCTIONS

Date	Reductions, in Feet, at:		
	Tygart	Lock 5	Pittsburgh
Oct., 1937	28.0	3.8	1.6
Dec., 1937	0.1 ±	1.1	0.5
Jan., 1939	8.0	2.2	0.8
Feb., 1939	24.0	3.0	0.6
April, 1939	26.0	3.8	1.8
March, 1940	4.2	0.6	....

Although the general principles of operating a flood-control reservoir are applicable to any watershed, the details of the operating plan must usually depend upon the characteristic features and requirements of each individual basin. Among the peculiarities to be considered in relation to the Tygart Reservoir are its location in the headwaters, the areas susceptible to flood damage at Pittsburgh and in the Monongahela Valley, the existing navigation on the river, the low spillway capacity, and the central location of Dam 5. The stage at Dam 5 is utilized as a control for the release of flood storage. These factors are used in developing the plan of operation of Tygart Reservoir but of course would not be available on any other watershed. On other streams and for other reservoirs a different set of characteristics necessarily would be considered.

Usually the large industrial centers like Pittsburgh that are subject to heavy flood loss would be the controlling point for reservoir operation, but the authors have shown that navigation is of such importance on the Monongahela River that a control point near the center of the basin must be selected. It appears to the writer, however, that Pittsburgh should be considered as a secondary control point for operations in the event of a general flood covering the Allegheny Basin.

A storage reservoir can reduce downstream flood crests by two means: (1) Storage until the flood has subsided throughout the basin, or (2) by delaying the peak at the site until the flood crest of the valley below has moved downstream. The latter plan is uniquely available to a reservoir in the headwaters. It has been used effectively in the operation of Tygart Reservoir as may be seen in the hydrograph of the flood of December, 1937; practically no reduction of peak was obtained at the site, where it was merely delayed, but a reduction of 1.1 ft was achieved at Lock 5. The floods of 1939 likewise show the effect of regulation to delay the peak as well as to reduce it by temporary storage.

The reduction of peaks on the main stem, by a delaying operation at the reservoir, is dependent to a considerable extent upon the location of the reservoir with respect to the main stem. Tygart Reservoir, being situated in the headwaters of the main stem, is favorably located for such operation. In order to illustrate a contrary condition, assume that floods on the Youghiogheny River under natural conditions discharge into the Monongahela River slightly ahead of the crest on the main stem. Under this condition a small reservoir located near the mouth of the Youghiogheny River, unable to hold a substantial part of the flood runoff, would succeed only in delaying the tributary crest, cause it to coincide more closely with the peak on the main stem, and thereby produce a greater flood below the mouth of the tributary. The location of a reservoir near the mouth of the Youghiogheny River and the relative time of the tributary flood is suppositive as far as the writer is aware, but it serves to illustrate the fact that merely delaying a flood crest on a tributary is not always advantageous, although it operates favorably for flood control by Tygart Reservoir.

A flood on a river is not a simple summation of excessive areal runoff as some theoretical studies may lead one to believe, but it is a compound product of the various peaks of its tributaries that combine at different times and places. The "flood-wave" analogy should not be taken too literally.

As another aspect of reservoir operation, the authors emphasize the element of flood prediction both above and below the reservoir. This emphasis is rightly placed, for some measure of discharge prediction at the reservoir and stage prediction at the control point is essential for efficient operation. Such flood prediction should be obtained with the closest possible cooperation with the Weather Bureau and the Water Resources Branch of the U. S. Geological Survey, such as the authors indicate is accomplished. The writer believes that snow surveys should also be utilized to secure data of the moisture content of snow cover if there is any possibility that such cover can contribute an appreciable proportion of the flood runoff.

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DISCUSSIONS

COMPACTION OF COHESIONLESS FOUNDATION SOILS BY EXPLOSIVES

Discussion

BY MESSRS. R. G. HENNES, AND JOHN D. WATSON

R. G. HENNES,[5] ASSOC. M. AM. SOC. C. E.[5a]—Many engineers have emphasized the importance of making generally available the results of experience with foundations and earthwork if the principles of soil mechanics are to be developed and utilized to best advantage. As a significant contribution to this field, the present paper should be most welcome. The value of such contributions is not fully exploited except by discussion, which preferably should spring from a background of comparable experience in the field. Description of new construction methods is unavoidably handicapped by the absence of parallel experience upon which adequate discussion could be based. It is to be hoped, therefore, that the author's closure of the present paper will be made the vehicle for presenting further information on this new method of compaction for the use of the profession.

Especially appreciated would be any additional test data that may be available which would assist the engineer in applying this method to other projects without unnecessary duplication of effort. In any such application the main thing to be determined is the size and distribution of charges. The variables upon which the layout depends are stated to be the type of soil, its degree of saturation, and the depth of the deposit. Fortunately, the problem is somewhat limited by the practical consideration that only in loose sandy deposits does the critical void ratio tend to be a controlling factor. Theory would seem to approve the author's conclusion that best results are to be expected when the soil is saturated; for the very danger associated with the concept of a critical void ratio is the "liquefaction" produced by the excess of water in the voids. The Denison tests suggest that the effect of saturation is less simple, and point to the need for further research in this field. The effect of depth is something that can be disclosed only by experiment, and it is here that the paper comes closest to permitting at least tentative general conclusions.

NOTE.—This paper by A. K. B. Lyman, M. Am. Soc. C. E., was published in May, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1941, by Reuben M. Haines, Assoc. M. Am. Soc. C. E.

[5] Asst. Prof., Civ. Eng., Univ. of Washington, Seattle, Wash.

[5a] Received by the Secretary November 27, 1941.

It would be helpful if the author's closure considered this matter at greater length. Both at Franklin Falls and at Denison best results appear to have been obtained by placing charges at a depth of 15 ft, but in neither case is it clear how deep the effect was felt, and it would be of interest to learn what evidence led to the conclusion "that where loose strata of sand greater than 30 ft thick are to be compacted, two or more tiers of small charges are to be preferred to one tier of large charges" (see "Introduction").

It is not the thought of the writer that any amount of discussion can obviate the need for preliminary tests on each individual project, but rather that each new investigation should be shaped by the fullest comprehension of what has already been accomplished.

JOHN D. WATSON,[6] ASSOC. M. AM. SOC. C. E.[6a]—The results presented in this paper show positively that the method is economical and that loose cohesionless soils can be made much denser, and also considerably less permeable. Colonel Lyman is to be commended most heartily for originating and developing a method of foundation improvement that will be of great value to foundation engineers throughout the world.

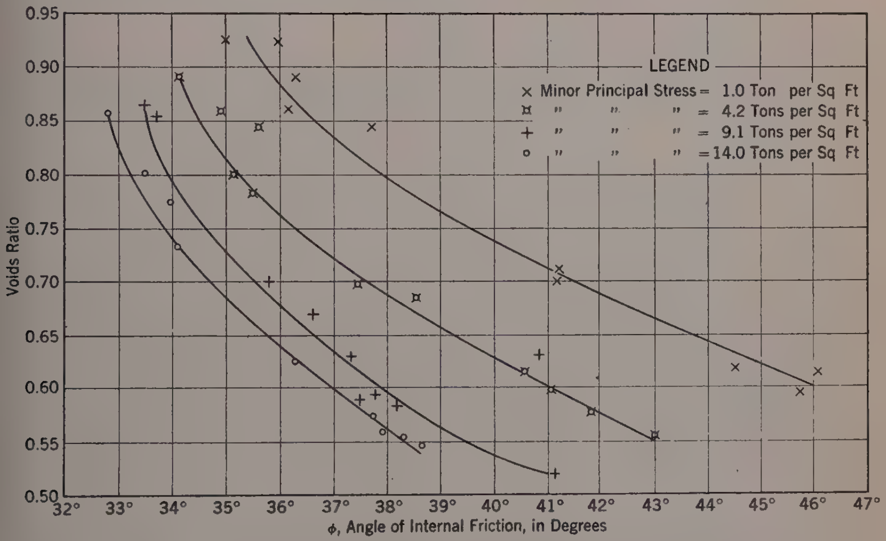

FIG. 13.—EMPIRICAL RELATIONSHIP BETWEEN VOIDS RATIO AND ANGLE OF INTERNAL FRICTION FOR FRANKLIN FALLS SAND

Although the author states (see "Introduction") "that the stability of the embankment would be considerably increased if these loose sand deposits could be consolidated to reduce the possibility of their failure by liquefaction * * *," he did not take the opportunity to point out the concomitant advantage of the greater resistance to rupture that these soils would have by reason of the higher density. In 1938, when the writer was serving as research fellow in soil mechanics for the Graduate School of Engineering at Harvard University, in Cambridge, Mass., he made many tri-axial compression tests on samples of a fine cohesionless sand from the Franklin Falls Dam site. The samples for these tests were prepared with a wide range of voids ratio (0.925 to 0.52, comparable to a unit dry weight of 88 lb per cu ft to 112 lb per cu ft), and the tests were made with a surrounding hydrostatic pressure ranging from 1.0 to 14.0 tons per sq ft. The results of these tests are shown in Fig. 13. For the same minor principal stress (hydrostatic pressure), the angle of internal friction is inversely proportional to the voids ratio. However, the amount of the variation changed with the intensity of the minor principal stress. When the minor principal stress was low, the increase in the angle of internal friction with a decrease in the voids ratio was greater than when the minor principal stress was high. The following explanation for this fact has been offered tentatively: When the minor principal stress is low, rupture occurs by one grain of sand sliding over another, and the interlocking effect of the grains is considerable. When the minor principal stress (and the major principal stress as well) is high, then many grains in the plane of failure break apart and the interlocking effect is less pronounced. Similar variations in the angle of internal friction with changes in density and minor principal stress were found for other sands.

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[6] Asst. Prof., Civ. Eng., College of Eng., Duke Univ., Durham, N. C.

[6a] Received by the Secretary December 15, 1941.

Karl Terzaghi, M. Am. Soc. C. E., has pointed out many times that the strength of a cohesionless soil foundation does not depend upon the first, but rather upon the fourth, power of tan $\left(45 + \frac{\phi}{2}\right)$, in which ϕ is the angle of internal friction. Hence, when the angle of internal friction of any sand is raised by compaction from 34° to 38°, the resistance to rupture of this soil beneath a foundation load increases not just 9%, but rather it increases 42%; and, if the angle of internal friction could be raised from 34° to 46°, the strength of the material against rupture by a foundation load would increase 200%. Although the possibility of loading a foundation of cohesionless soil to the point of rupture by means of an embankment is admittedly remote, nevertheless the large increase in the strength of such soils through compaction is quite pertinent for other types of foundation loading and for the stability of embankments as well.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

TRAFFIC ENGINEERING AS APPLIED TO RURAL HIGHWAYS

Discussion

BY CHARLES M. NOBLE, M. AM. SOC. C. E.

CHARLES M. NOBLE,[12] M. AM. SOC. C. E.[12a]—The modern attitude of the more progressive group of highway engineers is reflected in this paper in that the functional character of the highway in relation to the operation of the vehicle and the characteristics of the driver is stressed. The writer is strongly sympathetic to this view and therefore wishes to strengthen that type of thinking and indorse the paper.

It is encouraging to note the reference to research in traffic (driver) psychology, for it is believed[13] that such research will yield substantial dividends in transportation efficiency and accident reduction by pointing the way to correct design.

In addition to the principle enunciated by the author (that it may be advisable to introduce into the highway structure a physical change that will convert one type of accident into another less dangerous), it is also quite possible for the designer of a new facility to visualize many accident possibilities in advance, during the design process, and to arrange the design in anticipation of such possibilities. This is illustrated in the case of the Pennsylvania Turnpike tunnels, where it was necessary to converge from two lanes into one lane on entering, and the possibility of a skid or other mishap occurring in the confined roadway in front of the portals was anticipated. With a conventionally designed portal such a mishap, in many cases, would result in a direct collision with the immovable portal structure, generally with serious consequences. In the Turnpike design, a wall of the stepped type was projected out from the portal face, curving away from the roadway gradually until it disappeared into the bank of the approach cut. This wall was designed for the purpose of converting a direct impact accident into one of less serious

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NOTE.—This paper by Milton Harris, Assoc. M. Am. Soc. C. E., was published in May, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by Charles E. Conover, M. Am. Soc. C. E.; October, 1941, by Stephen E. Butterfield, Assoc. M. Am. Soc. C. E.; and December, 1941, by Messrs. Nathan Cherniack, and Park H. Martin.

[12] Chf. Highway Engr., War Dept., Arlington, Va.

[12a] Received by the Secretary November 28, 1941.

[13] *Proceedings*, Highway Research Board, 1937, p. 246.

character of the sliding type. Actual operation experience indicates that the walls are functioning with complete success, and many serious accidents have been averted by their use.

From the standpoint of transportation efficiency, the analysis by the author, utilizing a virtual speed profile to determine time losses, is valuable; for, although the average driver would be convinced immediately that an improvement should be made, it is often difficult to convince highway authorities, struggling with budgetary problems, of the necessity for improving part of an existing highway. The method illustrates the critical locations graphically and provides an economic argument that may be used to convince the authorities that funds should be authorized for remedial measures.

The author states (see heading "Traffic Data for Design") that "Design might be briefly described as the accommodation of structure to facts"; and

> "Only in late years has he [the highway engineer] paused for an instant to realize that the very element of his transportation problem—traffic—for which he has been building roads has not only kept up with the rate of production but has so far exceeded it as to create a problem of its own. Fundamental to design is knowledge of the traffic that will use the structure; therefore all traffic facts possible should be gathered for use in design."

He also states that "As more thought is given to this subject, so the field will expand and more and better information will be demanded and obtained; and its analysis will be more definite." All these statements should be applauded. Again the author goes to the heart of the modern traffic problem in the paragraph beginning "The peak-hour traffic is usually the criterion for which the designer plans."

With respect to the author's comments relative to design speed, it is pertinent to remark that, if too low a value for design speed is selected for the main routes, such highways may again become obsolete within a relatively short period (10 to 15 years), resulting in an economic loss, and the engineer may suffer embarrassment and possible loss in prestige in having to acknowledge the adoption of a shortsighted policy.

In closing, a word of caution should be interjected, for there is a tendency in some quarters to feel that engineering can solve all traffic problems, human and otherwise, and that enforcement can be relaxed when modern design is provided. Such is far from the case, as Mr. Harris recognizes clearly, and the writer wishes to emphasize the point that modern highway engineering and enforcement should function in unison.

Mr. Harris has covered his subject quite thoroughly as far as general principles are concerned, and very little can be added except as a matter of indorsement.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PILE-DRIVING FORMULAS

PROGRESS REPORT OF THE COMMITTEE ON THE BEARING VALUE OF PILE FOUNDATIONS

Discussion

BY MESSRS. LEWIS C. WILCOXEN, H. A. MOHR, AND A. E. CUMMINGS

LEWIS C. WILCOXEN,[50] ASSOC. M. AM. SOC. C. E.[50a]—The Committee's hope "that all persons who can possibly do so will contribute to this discussion, in the hope that a clearer understanding will result," prompts the writer to call attention to a peculiarity of the pile-bearing formula that he developed some years ago.[51] It was only when this formula was compared with Eq. 3 of the Committee's Report that this peculiarity became apparent, and it is of such importance that he wishes to call attention to it.

The formula was derived empirically by means of driving a model wood pile 1 in. square in cross section and 6 in. in length, with lead weights of 1.0 lb and 0.1 lb. The former was dropped from a height of 1.0 ft, and the latter from the successive heights of 1.0 ft, 0.1 ft, and 0.01 ft, yielding respectively the wide range of impacts of 1.00, 0.10, 0.010, and 0.001 ft-lb.

The piles were driven into six soils—three of sand with different degrees of compaction and three of clays with various degrees of plasticity. They were driven into the soils by observed static loads, and then the drive under the various impacts was noted. The observed results were plotted on log log paper with $\frac{W\,h}{R_d}$ and s as the vertical and horizontal scales. The resulting points were grouped closely along straight lines, proving the formula $R_d = \frac{W\,h}{S^x}$, in

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NOTE.—This Report was published in May, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1941, by Messrs. G. G. Greulich, C. O. Emerson and D. O. Northrup, Harry J. Engel, and John D. Watson; October, 1941, by Messrs. Robert D. Chellis, Lazarus White, John G. Mason, Carlton S. Proctor, George Paaswell, and Abraham Woolf; November, 1941, by Messrs. Howard T. Evans, William G. Atwood, Donald M. Burmister, Wallace E. Belcher, Clement C. Williams, and D. P. Krynine; and December, 1941, by Messrs. Trent R. Dames and William W. Moore, Maxwell M. Upson, Gregory P. Tschebotarioff, Robert F. Legget, and Jacob Feld.

[50] Asst. Civ. Engr., City Engr.'s Office, Detroit, Mich.

[50a] Received by the Secretary November 26, 1941.

[51] "New Pile-Bearing Formula from Model-Pile Tests," by Lewis C. Wilcoxen, *Engineering News-Record*, November 3, 1932, pp. 524–526; also, discussion, *ibid.*, March 16, 1933, p. 355.

which the value of x ranged from 0.6 for the soft clay to 1.0 for the most compact sand. The writer believes that this demonstrated formula merits careful study.

The value of R_d in Eq. 3 was equated to the writer's formula from which was derived:

$$S^x = \frac{s + k}{e\left[1 - \frac{P(1 - n^2)}{W + P}\right]} \quad \text{(48)}$$

In Eq. 48, S = total settlement, in feet, and x is a constant for each particular soil. Since in some of the writer's experiments all the factors of this equation were constant except S, it is impossible by means of it to explain the different proved values of x. The quantity x must necessarily be an important soil characteristic coefficient.

The writer holds no brief for pile impact formulas. Nevertheless, he would add that until field-test results are checked against formulas including a proper soil factor, investigators are not warranted in denying the possibility of developing a practical one.

The writer's experiments were deficient in so far as they ignored the factors e, n, and p. If these factors are important, then his formula is incomplete. Regardless of its possible deficiencies, it does include the important soil characteristic factor, the omission of which in turn makes Eq. 3, and all formulas derived from it, incomplete.

H. A. Mohr,[52] M. Am. Soc. C. E.[52a]—After studying the formulas derived in Report *A* and "worrying" through Mr. Hiley's published work,[1] upon which analysis the proposed formulas are based, it is the writer's firm conviction that their inclusion in the proposed Manual would be a grave mistake.

That analysis is quite detailed and reads well, but still it is theory, and the conclusions otherwise are based upon a paucity of practical data. Answers obtained by its use are no more consistent and logical than those obtained by the use of other formulas. Its only obvious advantage to those who wish to be critical of present formulas is the great number of unknowns to which a series of values may be applied until an answer satisfactory to the interested party is finally reached.

Paragraph *A*-4 of Report *A* reads "Every pile-driving formula may be derived from the general formula (Eq. 3) by making certain assumptions regarding the values of the various factors," which is interesting. For years (some historian can name them) since the first pile formula was introduced, engineers and others have discussed the issues and argued about the validity of assumptions used in deriving the remainder of the formulas extant. Now it is proposed that the Society, when publishing the Manual, give its blessing to still other formulas, which in no respect are more generally applicable or accurate than existing formulas.

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[52] Waban, Mass.

[52a] Received by the Secretary December 4, 1941.

[1] "Pile-Driving Calculations with Notes on Driving Forces and Ground Resistance," by A. Hiley (Theory and practice; table of forces transmitted through pile; energy requirements; bearing qualities of ground), *Structural Engineer*, Vol. 8, 1930, pp. 246–259, 278–288.

Substituting the values assigned to the "various factors" to simplify the final form is merely compounding assumption, because the values are assumed. If the profession is satisfied to continue this procedure, it can rest assured that the field of assumptions is so broad as to plague it with the problem to eternity.

Consider the field for assumptions: Types of piles, shapes of piles, sizes of piles, materials of which piles are made; variety of hammers used, methods of installation, types of equipment for handling and driving piles, job conditions, weather conditions, condition of equipment, types of soil and combinations of soil types into which piles are driven, types of structures to be supported, etc.

Obviously no pile-driving formula is adequate for these variable conditions. Besides, if a perfect formula could be derived its answer would not necessarily provide a satisfactory pile foundation. So one wonders, why all the fuss and desire for change? The change, if any, should start with other than the laws of impact as a basis. Engineers are "getting nowhere fast" by inserting new assumptions, ever so often, into a basic formula that has been worn thin. No facts are available to prove "that Eqs. 8, 9, 16, and 17 are based on reasonable assumptions * * *"; nor can any one delimit "* * * cases within the validity of the assumptions" as is intimated in Paragraph *A*-12. What would seem reasonable to one party may seem unreasonable to another and still both parties may be wrong in fact—all of which results in confusion at its worst.

Then, why insert a static formula? Values of "*f*," to have any meaning, must be established from load tests to failure, and reliable tests of that type have been very, very few. What would be a safe value of "*f*" for one location may well be totally unsafe for another location. It is true that safe pile load under limiting conditions may be based upon skin-friction value, where a safe value has been established by practical experience. To suggest the inclusion of this formula and its meaningless Table 1 is scarcely enlightening.

The science of soil mechanics will not produce a static pile formula of any greater accuracy for universal application than present dynamic formulas because those same variables are inherent in the solution of any pile-formula problem. One needs but little practical experience in pile driving to know that radical changes in pile lengths occur on many jobs and in many instances even within small pile footings.

Before it can function at all, the science of soil mechanics requires complete subsurface data and samples of soil for identification and laboratory tests. This is a common-sense approach to every foundation problem except that the laboratory test is superfluous in such a high percentage of cases that it is necessary only to acknowledge its necessity in special problems. That procedure has not been common practice in the past, and although improvement in that direction has been made in recent years, it is far from universal practice at present. Determination of the proper type of foundation to use on any other basis is ineffectual. Consequently, pile foundations have been resorted to for no other reason than that their use provided an easy solution, a cure-all or an "out" in many instances where they may not have been needed and the type of pile naturally could not have been selected to best suit existing ground conditions. Even where the use of piles is a proper solution, with no data on soil conditions at the site, it is impossible to know that they will function as expected, although they are driven to satisfy every pile formula that has been developed.

Instances confirming this practice occur so constantly as to cause wonderment. At the same time it should be emphasized that the same unintelligent procedure results in unsatisfactory foundations of other types than piles wherein no driving formula enters as a factor of construction.

It would be interesting to learn just how any pile formula will save that situation. Also it would be interesting to learn of one instance in which the requirements of the "Engineering News" "steam hammer" formula denominator $(s + 0.1)$ can be proved the sole cause of a pile foundation not performing as expected. There is no intention here to belittle the extent of troublesome foundation performance by merely requesting that single proof.

Experience has proved, however, that the use of the "Engineering News" "drop hammer" formula denominator $(s + 1.0)$ has resulted in complete destruction and mangling of many wood piles, this being the type of pile most extensively driven with drop hammers.

Theoretical refinements probably have less justification in the field of pile foundations than in any other department of technical design and construction. So, rather than guess at a variety of coefficients, would it not be preferable to judge the result by experience in the first place and do away with a complicated mathematical process to obtain a questionable answer?

Since engineers and others will use a pile formula of some kind, it seems that the "Engineering News" formula denominator $(s + 0.1)$ should be inserted in Report *B* and its limitations stated. It has been established in the United States, at least, by years of common usage, with no record of inadequacy against it, when properly used.

Other sections of the Manual must clarify the reference to test piles mentioned in both Reports *A* and *B*.

A. E. CUMMINGS,[53] M. AM. SOC. C. E.[53a]—In the writer's opinion, the publication of Report *A* in a Manual of Engineering Practice would be a serious mistake. All of the formulas given in the Report were published at least fifty years ago and engineers have been "tinkering" with them ever since. The usual procedure is to make one assumption after another; to retain some terms and cast out others; and then to publish a "new" pile-driving formula. For example, Eq. 4 is said to have been proposed by Hiley in 1930. Actually, this equation is nothing more than the so-called "complete" pile-driving formula published by Redtenbacher in 1859.[54]

This is readily demonstrated by means of a few substitutions and a little elementary algebra. If the expression $\frac{R_d\, L}{2\, A\, E}$ from Eq. 10 is substituted for k in Eq. 4, and if $n = 0$ and $e = 1.0$ are also substituted in Eq. 4, the resulting quadratic equation is easily solved for R_d and the solution is Eq. 19. In other words, Eqs. 4 and 19 are based on the same set of fundamental assumptions in spite of the fact that they do not look alike as published in the Report.

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[53] Dist. Mgr., Raymond Concrete Pile Co., Chicago, Ill.

[53a] Received by the Secretary December 5, 1941.

[54] "Prinzipien der Mechanik und des Maschinenbaues," by F. Redtenbacher, 1859.

There are only five basic types of dynamic pile-driving formulas in use at the present time and all of them can be represented by the formula

$$W\,h = R_d\,s + Q \qquad (49)$$

in which Q represents all the energy losses that occur during impact. For many years engineers have been making all kinds of assumptions as to what should and what should not be included in Q. The profusion of pile-driving formulas that can be found in engineering literature is simply the result of these assumptions.

The simpiest formulas are the empirical formulas such as the "Engineering News" formula which can be expressed as

$$W\,h = R_d(s + 0.1) = R_d\,s + 0.1\,R_d \qquad (50)$$

When he published this formula, the late A. M. Wellington,[55] M. Am. Soc. C. E., objected to having it referred to as an empirical formula. However, the development of the formula was based on experience rather than on theoretical mechanics and such formulas are rightly called empirical. The energy loss deduction amounting to 0.1 R_d was determined from a work diagram representing the behavior of the pile under one blow of the hammer.

Long before the "Engineering News" formula was published all of the other basic types were well known. As a matter of fact, it was the erratic results indicated by all of the then existing formulas that caused Wellington and others to reconsider the entire problem of pile-driving dynamics fifty years ago. Very few engineers appear to be aware of this fact at the present time. George L. Freeman,[56] M. Am. Soc. C. E., has described the "Engineering News" formula as a "hoary old" formula. Actually, the "Engineering News" formula is the youngest of the lot and it came into existence because of the inability of the other formulas to give consistent results.

Eq. 20 can be written in the basic form as

$$W\,h = R_d\,s + W\,h_0 \qquad (51)$$

in which the energy loss is represented by $W\,h_0$. This formula is erroneously attributed to Franz Kreuter in the Report.[3] A formula of this type was first published by G. J. Morrison[57] in 1868 so that it antedates the "Engineering News" formula by at least twenty years. Eq. 51 has been the subject of considerable investigation, but the results obtained were quite erratic. Paragraph *A*-10 refers to a study of this formula that was made at Boulogne-sur-Mer, although the detailed record of the experiments is not given. These details are readily available elsewhere,[58] but they fail to substantiate the reliability of this formula. Four piles were tested and all of them were driven into the same kind of soil with the same drop hammer. Three of the piles were 35 ft long with 16-in. butts and each weighed about 2,700 lb. The fourth pile was 30 ft long with a 12-in. butt and it weighed about 1,400 lb. According to Paragraph *A*-10, the value of h_0 determined in the field was the same for all four piles. According to Paragraph *A*-9, "the value of h_0 varies with the weight * * *, area, and length of the pile." It is difficult to understand why the small pile gave the same value of h_0 as the three large ones.

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[55] *Transactions*, Am. Soc. C. E., Vol. XXVII (1892), p. 129.

[56] "A Practicing Engineer Looks at Soil Mechanics," by George L. Freeman, *Civil Engineering*, December, 1938, p. 811.

[3] "A New Method for Determining the Supporting Power of Piles," by Franz Kreuter (Mathematical formulas for wooden piles), *Minutes of Proceedings*, Inst. C. E., Vol. 124, 1895–1896, Pt. 2, p. 373. Abstract in *Railway Review*, Vol. 36, May 9, 1896, p. 262.

[57] *Minutes of Proceedings*, Inst. C. E. (London), Vol. 27, 1868, p. 313.

[58] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Vol. II, 1936, p. 219.

A much more elaborate investigation of Eq. 51 was made by Ph. Krapf[59] on a considerable number of wood piles driven along one of the German canals. Krapf was unable to obtain consistent values of h_0 and he found it necessary to introduce another coefficient into the formula. The introduction of a coefficient to make a formula check a set of experiments simply means that the theoretical basis of the formula is unsound and the empirical coefficient changes the theoretical formula into an empirical formula.

The theoretical analysis leading up to Eqs. 16 and 17 involves several questionable assumptions although there is nothing to indicate this in the Report. Furthermore, the practical use of the h_0-method is limited to drop hammers, and very little pile driving is being done with drop hammers at the present time. To vary the length of the stroke of a steam hammer, it is necessary to do a machine shop job on the hammer.

The third basic type of dynamic pile-driving formula may be written in the form

$$W\,h = R_d\,s + \frac{R^2_d\,L}{2\,A\,E} \qquad (52)$$

When this quadratic equation is solved for R_d, it becomes the Weisbach formula which was published about 1850:

$$R_d = -\frac{s\,A\,E}{L} + \sqrt{\frac{2\,W\,h\,A\,E}{L} + \left(\frac{s\,A\,E}{L}\right)^2} \qquad (53)$$

In formulas of this type, the temporary elastic compression of the pile is assumed to be the only energy loss that needs to be considered. Certainly there are other energy losses involved in pile driving but Eq. 52 ignores these. Furthermore, the equation is based on the assumption that the dynamic energy loss can be computed by static theory. The last term in Eq. 52 represents the potential energy of strain in a compressed strut subjected to a static load of amount R_d at each end. In the first place, it is well known that the elastic compression under impact is something entirely different from the elastic compression due to a static force. In the second place, the resistance is scarcely ever applied entirely at the pile point. There is usually some resistance along the sides of the pile and in many cases practically all of the resistance is on the sides and the point resistance is negligible. Accordingly, it is not to be expected that the energy loss due to temporary elastic compression can be computed with any reasonable degree of accuracy by means of an expression taken from static theory without modification for use in a dynamic problem.

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[59] "Formeln und Versuche über die Tragfähigkeit eingerammter Pfähle," by Ph. Krapf, *Fortschritte der Ingenieurwissenschaft*, Group 2, No. 12, Leipzig, 1906.

The fourth basic type of dynamic pile-driving formula may be written in the form

$$W\,h = R_d\,s + W\,h\,\frac{P(1 - n^2)}{W + P} \qquad (54)$$

In this equation, the last term represents the energy loss on the assumption that pile driving is an impact problem that can be solved on the basis of the impact theory established by Newton. When the impact is assumed to be perfectly inelastic ($n = 0$), Eq. 54 reduces to

$$R_d = \frac{W\,h}{s\left(1 + \frac{P}{W}\right)} \qquad (55)$$

which is the pile-driving formula published by Eytelwein in 1820.

As to whether or not pile driving is a problem in Newtonian impact, the most reliable authority should be Newton himself. After describing his experiments, Newton[60] states that his method can be applied to various kinds of elastic bodies

> "* * * except where parts of the bodies are damaged in the collision or where they suffer some such extension as occurs under the strokes of a hammer."

The first part of Newton's statement has come down in the form of the usual specification requirement that the broomed heads of wood piles must be sawed off before the final hammer blows are struck. The significance of the second part of Newton's statement is being overlooked by engineers who think that pile driving is a problem in Newtonian impact. In order to understand this it is only necessary to consider the fundamental difference between pile driving and the experiments Newton made in the development of his impact theory.

Newton's impact experiments were made with spheres suspended as pendulums. When the spheres were at rest they were just tangent to one another. One sphere was then pulled away and allowed to swing in an arc and strike the stationary sphere. The movements of both spheres after impact were carefully measured. These spheres were free bodies except for the restraints produced by the strings on which they were suspended. However, these restraints affected only the path in space which the spheres had to follow and there were no restraints that could contribute to the elastic distortions of the spheres themselves. In the pile-driving problem the movement of the pile is hindered by external reactions from the surrounding earth and the conditions are not the same as those under which Newton made his experiments with spheres. Newton deduced his impact theory as a part of the proof of his third law of motion. In his explanation of this law, Newton[61] mentions two colliding bodies and gives his rules for their behavior provided that "* * * the bodies are not hindered by any other impediments."

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[60] "Philosophiae Naturalis Principia Mathematica," by I. Newton, 3d Ed., 1726, Scholium to Corollary VI, p. 24.

[61] *Ibid.*, Law III, p. 14.

The significance of this question of external restraint is readily explained by an analogy from the game of billiards. When the cue ball collides centrally with the object ball in the middle of the table, the subsequent behavior of the two balls is determined almost entirely by the coefficient of restitution for ivory on ivory. If the object ball is frozen against a cushion and if the cue ball is driven against it at a right angle to the cushion, the subsequent behavior of the balls is no longer controlled by the coefficient of restitution for ivory on ivory. The restraint provided by the cushion and the elastic properties of the cushion become factors in the problem and an impact phenomenon of this kind is a three-body problem. Newton's impact theory with its coefficient of restitution is limited to two-body problems.

The most simple kind of pile-driving operation is the case in which a drop hammer strikes the unprotected head of a wood pile. Even in this simple case, the Newtonian theory of impact cannot be used because of the restraints applied to the pile by the surrounding soil. In most pile-driving operations there are several objects such as a driving bonnet, a cushion block and several steel plates between the pile head and the hammer. In addition, there is the restraint of the surrounding earth. The pile-driving problem is therefore entirely outside the scope of the elementary Newtonian theory of impact and there is no such thing in pile driving as a "coefficient of restitution" in the Newtonian sense.

The so-called "complete" pile-driving formula is the fifth basic type and this may be written in the form:

$$W\,h = R_d\,s + \left[W\,h\,\frac{P(1-n^2)}{W+P} \right] + \left\{ \frac{R^2_d\,L}{2\,A\,E} + \frac{R^2_d\,L'}{2\,A'\,E'} + K \right\} \ldots (56)$$

In this equation, L', A', and E' refer to the driving cap, and K is the energy loss due to the temporary elastic compression of the soil. The two bracketed terms on the right-hand side represent the total energy loss. It is easily seen that this is Eq. 3 when k in Eq. 3 is replaced by its value as given in Eq. 11.

The term in square brackets is the energy loss derived on the assumption that pile driving is a problem in Newtonian impact. The use of this term in a pile-driving formula is based on a misunderstanding of the scope of Newton's theory of impact. The term in curled brackets represents the energy loss due to the temporary elastic compression of the pile, the pile cap, and the soil. The expressions included in this term are based on the assumption that there is practically no difference between dynamics and statics. It is assumed that the temporary elastic compression under impact can be computed by a static formula. In other words, Eq. 56 includes all the erroneous assumptions that are involved in Eqs. 52 and 54.

An even more serious fallacy involved in Eq. 56 is the inclusion of those two bracketed terms in a single equation. In Newton's experiments, both spheres were elastically distorted during the collision and a small amount of heat was generated. However, Newton did not attempt to analyze the problem by computing elastic distortions or any other "particular" kind of energy losses. He based his theory of impact on what is now called the coefficient of restitution and, by definition, the coefficient of restitution includes "all" of the energy losses that occur in a given case of Newtonian impact. The term in curled brackets in Eq. 56 refers to particular energy losses in the form of elastic distortions but these are already included in the Newtonian term. Accordingly, the term in square brackets and the term in curled brackets are mutually exclusive and one or the other of them should be eliminated. When both terms are included, some of the energy losses are being subtracted twice.

The fact that there are excessive energy loss deductions in pile-driving formulas like Eq. 9 can be demonstrated in several ways. In the first place, there is the question of the safety factor. Practically all of the other basic types of formulas are used with a safety factor of 6. For Eq. 9, a safety factor of 3 is recommended. Actually, the formula itself is seriously erroneous on the side of safety so that a small safety factor is necessary to compensate for the error. If the usual safety factor of 6 were used, the formula would require absurdly high driving resistances, and any engineer familiar with actual pile-driving operations would know that there was something wrong with the formula. In the second place, Eq. 9 does lead to absurd results in some cases in spite of its small safety factor. The piles used for the foundations of the James River Bridge[62] were pre-cast concrete piles 24 in. square and 115 ft long. They were driven with a single-acting steam hammer having a ram weighing 7,500 lb and a stroke of 42 in. It was required to drive them to an indicated safe bearing capacity of 40 tons each. To apply Eq. 9 to this problem, it is necessary to solve the equation for the required penetration

$$s = \frac{3.6\,W\,h}{R} \times \frac{W}{W + P} - k \ldots\ldots\ldots\ldots\ldots\ldots (57)$$

If Young's modulus of concrete is taken as 3,000,000 lb per sq in. and if Eq. 11 is used to compute k, Eq. 57 gives a negative value of s. When this result is obtained from the formula, it means that a heavier hammer is required. Nevertheless, the piles were driven as described and the bridge is standing satisfactorily.

The other basic types of pile-driving formulas can be solved for s and applied to this James River Bridge problem. The driving resistances required by the different formulas for the 40-ton load will vary over a wide range, but none of them will lead to the conclusion that the job could not be done with a single-acting steam hammer with a 7,500-lb ram and a 42-in. stroke. Since these were very heavy piles, the instructions in Paragraph A-12 could be interpreted to mean that Eq. 4 should be used instead of Eq. 9 and that some numerical value of n should be substituted in Eq. 4. However, it appears as if an engineer would have a difficult problem in trying to decide when to use Eq. 9 with $n = 0$ and when to go back to Eq. 4 with some finite value of n. The process undoubtedly requires a certain amount of mental dexterity combined with a little *a posteriori* reasoning. If Eq. 9 gives an answer that does not fit the facts, it is only necessary to make a carefully selected set of new assumptions so that it will fit.

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[62] *Transactions*, Am. Soc. C. E., Vol. 95 (1931), p. 263.

The most unfortunate thing about Report *A* is the manner in which it presents the derivation of Eqs. 8 and 9. Whenever an assumption is made in the derivation of these equations, the assumption is said to be "reasonable" or "logical." Assumptions made in the derivation of other formulas are called "unwarranted." Eq. 4 is presented as an equation that "involves no simplifying assumptions." Actually, this equation involves assumptions that are fundamentally unsound from the standpoint of elementary mechanics. For example, Eq. 4 is based on the assumption that Newton's theory of impact with its coefficient of restitution can be applied to an impact problem involving more than two bodies. The error in this assumption is not a matter of opinion. It is a matter of fact which was stated clearly by Newton himself several hundred years ago.

In the writer's opinion, a Manual of Engineering Practice should not present technical information in the manner in which these pile-driving formulas are presented in Report *A*. Eqs. 8 and 9 are described as being reliable and of comparatively recent origin. As far as reliability is concerned, there is an abundance of field evidence available to show that such formulas are quite erratic. Furthermore, these formulas are not new since they were first published at least eighty years ago.

As a matter of fact, the only new concept that has been introduced into pile-driving dynamics in the past fifty years is the theory of the longitudinal impact of long elastic rods. This theory itself is not new as it was developed by St. Venant[63] and Boussinesq[64] many years ago. The application of the theory to pile-driving dynamics was first suggested by D. V. Isaacs,[65] and the British Building Research Board[66] in 1938 demonstrated the fact that the behavior of full-sized piles under actual field conditions can be predicted with considerable accuracy by means of this theory. The theory is concerned with the question of stress transmission through the pile and, unfortunately, it involves some rather difficult mathematics. However, there is a considerable amount of field evidence available which shows that the stress transmission characteristics of a pile are of great importance not only in determining its behavior during driving but also with respect to its subsequent ability to carry static load. This method of investigating the phenomena of pile-driving dynamics is one that deserves the careful attention of all engineers engaged in pile-driving work. It is a new and promising field for investigation.

Report *A* contains several other statements that should either be eliminated or presented in a different manner. In Paragraph *A*-12 is a statement about the "whipping" of light piles which is said to have a great effect on the computed dynamic resistance. It is a simple matter to analyze this problem to determine the effect of whipping. In the first place, most foundation piles are driven entirely into the ground and the pile-driving formula is used only at the end of the driving process. If such piles are driven through a surface crust of hard ground, they may whip at the beginning of driving. However, when the pile is in the ground and the driving resistance is being measured for the calculation of indicated bearing capacity, there is no whipping.

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[63] *Journal de Liouville*, Tome XII, 1867, p. 237.

[64] "Application des Potentiels," 1885, p. 508.

[65] *Journal of the Institution of Australian Engineers*, Vol. 3, 1931, p. 305.

[66] "An Investigation of the Stresses in Reinforced Concrete Piles During Driving," by W. H. Glanville, G. Grime, E. N. Fox, and W. W. Davies, *Technical Paper No. 20*, Dept. of Scientific and Industrial Research, Building Research Bd., London, 1938.

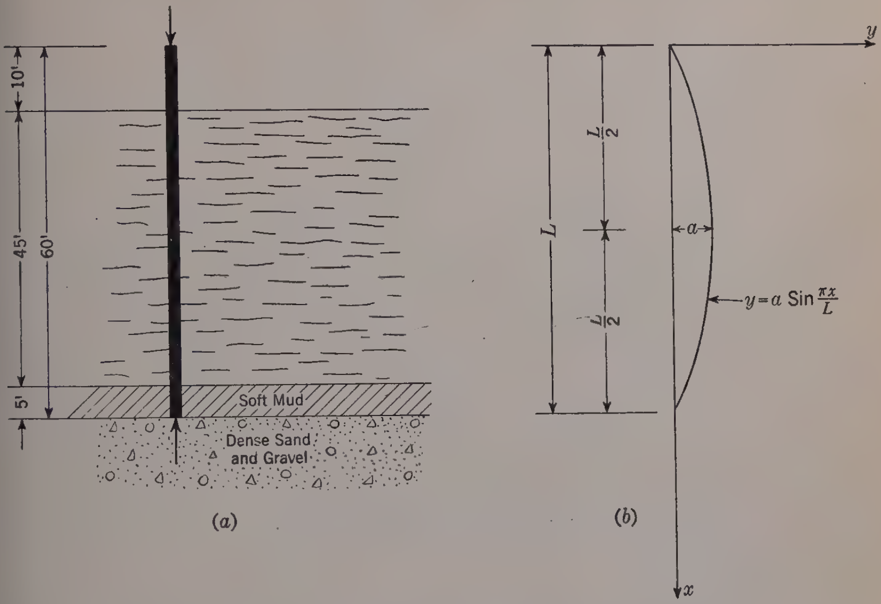

FIG. 10

In some cases, long slender piles are driven for marine structures under the conditions indicated in Fig. 10(*a*). Let it be assumed that the pile would bend into a single loop of a sine curve as shown in Fig. 10(*b*) so that the equation of the elastic curve could be expressed as

$$y = a \sin \frac{\pi x}{L} \qquad (58)$$

in which a is the maximum deflection at midlength. The strain energy, V, stored up in the bent pile can be determined from the equation

$$V = \frac{E I}{2} \int_0^L \left(\frac{d^2 y}{dx^2} \right)^2 dx \qquad (59)$$

in which E is Young's modulus of the pile material and I is the moment of inertia of the pile cross section. Differentiation of Eq. 58, substitution of the proper derivative in Eq. 59, and the subsequent integration of Eq. 59, will give the strain energy

$$V = \frac{a^2 \pi^4 E I}{L^3} \qquad (60)$$

The bending moment in the pile can be determined from the equation

$$M = - E I \frac{d^2 y}{dx^2} \qquad (61)$$

Differentiation of Eq. 58 and the substitution of $\frac{L}{2}$ in the derivative give the maximum bending moment

$$M_{\max} = \frac{a\,\pi^2\,E\,I}{L^2} \qquad (62)$$

The maximum fiber stress in the bent pile is determined from

$$\sigma_{\max} = \frac{M_{\max}}{S} \qquad (63)$$

in which S is the section modulus of the pile cross section.

Let it be assumed that a 10-in., 42-lb H-beam, 60 ft long, is being driven with a single-acting steam hammer having a ram weighing 5,000 lb and falling 3 ft. Let it be further assumed that the pile bends 10 in. out of line at mid-length under one stroke of the hammer. The quantities to be used in the computation are: $a = 10$ in.; $L = 720$ in.; $E = 30{,}000{,}000$ lb per sq in.; $I = 71.4$ in.,^{4} and $S = 14.2$ in.^{3} These last two quantities refer to the weak axis of the H-beam since the pile would most probably bend across that axis. Eq. 60 gives the energy loss as 14,100 in-lb, which is about 8% of the energy input of the hammer. Eq. 62 gives the maximum bending moment as 410,000 in-lb. Eq. 63 gives the maximum fiber stress as 29,000 lb per sq in., which is close to the yield point stress of 33,000 lb per sq in. commonly required for structural steel.

Accordingly, the pile can be bent to the yield point of the steel with the absorption of something less than 10% of the hammer energy. The same kind of calculation may be applied to wood piles and the results are similar. If the pile is shorter and stiffer or if the hammer is lighter, there is little or no whipping and consequently no serious energy loss from this cause. Although this method of calculation is only an approximate method, it indicates that the pile would very probably break before it could absorb more than 10% of the hammer energy.

Another part of Report *A* that needs revision is the section on static formulas which indicates that such formulas are practically useless. Actually, the problem of determining the bearing capacity of a pile is a static problem. During the past few years various investigators have studied this problem by means of static methods and the physical properties of the soils surrounding the pile. Relatively few of the test results have been published to the present time but it is the writer's considered opinion that the static approach to this problem has been neglected too long. Modern soil mechanics should be able to determine soil properties with sufficient accuracy to enable engineers to predict pile bearing capacities from static considerations. The possibility of doing this would require much careful investigation, but the static method represents a rational approach to a static problem that has been confused with dynamics for at least a century. Furthermore, the results obtained by static methods could scarcely be more erratic than the results now being obtained with dynamic formulas.

The erratic nature of the results obtained with dynamic formulas is a subject to which engineers have paid far too little attention. There is available a very considerable amount of pile-driving data from which it is possible to determine indicated bearing capacities by means of a number of dynamic formulas and then to compare these computed results with the actual bearing capacity determined by a load test to failure. When such data are tabulated it is always seen that some of the computed results are several hundred per cent above or below the actual test results. Many engineers tabulate data of this kind for a set of 25 or 30 experiments and then compute the numerical average of the test results apparently on the assumption that the numerical average is a figure with practical significance. Actually, the calculation of the numerical average is only the first step in the statistical analysis of a set of data of this kind. When the numerical average is compared with the individual test results, it is seen that only a few of the results are close to the average and that the remainder vary from the average as much as several hundred per cent in either direction. In the language of statistical mathematics the "deviations" of the individual results from the mean are very large and it is practically impossible to predict even the "most probable" value that could be expected in a given case.

It seems certain, therefore, that the problem of pile-driving dynamics includes factors that cannot be taken into account by formulas of the kind included in Report *A*. The complicated formulas are no more accurate than the simple ones although the complicated formulas may look authoritative. Accordingly, the pile-driving operation might just as well be controlled by a simple empirical formula. In regions where there exists a considerable amount of pile foundation experience, such empirical formulas have been developed and are being used. In regions where there is no previous pile foundation experience, the simple formula backed by good judgment and a careful soil investigation are as adequate to solve the problem as a complicated formula based on a half a dozen questionable assumptions.

As to how this material on pile formulas should be presented in a Manual of Engineering Practice, the writer has the following suggestions to make. The body of the Manual should contain a series of plain statements on the subject similar to Report *B*. If any mathematical derivations are to be included, they should be placed in an appendix. The mathematics should cover static as well as dynamic formulas and each formula should be accompanied by a clear statement of the assumptions on which it is based. Above all, the Manual should not present one formula as being thoroughly reliable and all others as being entirely unreliable.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

DESIGN AND CONSTRUCTION OF SAN GABRIEL DAM NO. 1

Discussion

BY JOSEPH JACOBS, M. AM. SOC. C. E.

JOSEPH JACOBS,[5] M. AM. SOC. C. E.[5a]—This is a splendid paper, presenting in commendable detail the procedures and technique of design and construction, and many of the results obtained in the actual building of a very important and somewhat unusual earth and rock-fill dam structure. It is desired to refer briefly to a few of the items discussed in the paper.

(1) *The Original Design.*—It is not clear why entirely adequate tests of quarry composition, as to the proportions of fine and coarse materials that would probably be yielded, were not made in advance of the preparation of plans and specifications. Presumably, more extensive tests than apparently were made would have avoided, or at least materially lessened, the train of unfortunate events (not fully discussed in the paper) that resulted from the lack of correct foreknowledge as to quarry composition. A dam structure of magnitude always justifies thorough and extensive advance exploration for required construction materials, foundation conditions, etc.

(2) It seems also, to the writer, that the specifications imposed too severe an exclusion of the fines, particularly in view of the fact that the rock was to be placed dry. The inclusion in the rock-fill section of a dam of a fair proportion of such fines, for example, as were eliminated by the original specifications, makes for a denser and more stable structure (even though the friction coefficient may be somewhat decreased) and one that is satisfactory as to permeability. In the present case, as was disclosed soon after construction was begun, the disparity in the proportions of coarse and fine material of Quarry 10 was too great to warrant construction, economically, of a rock-fill dam with borrow from that source.

(3) *The Revised Design.*—The assumption, or rather the crediting, of no sliding resistance to Zones 1 and 2 in determining dam stability was in the

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NOTE.—This paper by Paul Baumann, M. Am. Soc. C. E., was published in September, 1941, *Proceedings.* Discussion on this paper has appeared in *Proceedings,* as follows: November, 1941, by E. Soucek, Jun. Am. Soc. C. E.; and December, 1941, by William P. Creager, M. Am. Soc. C. E.

[5] Cons. Engr., Seattle Wash.

[5a] Received by the Secretary December 15, 1941.

direction of increasing the factor of safety against sliding failure, but that assumption may be regarded as too severe, for those zones definitely do have appreciable sliding resistance, particularly Zone 1 with its dominant rock content and with a shear coefficient probably not less than 0.75. An alternate, and perhaps a more economical procedure, would be to assign their nearest probable value to all zone resistances, rather than to ignore some of them as was done for Zones 1 and 2, and then apply a conservative over-all safety factor to the entire cross section of the dam, or to such portion of it as was involved in the plane or curved surface of least shear resistance. It is hoped that the author, in his closure, will indicate the location of the critical line of shear weakness and just how he obtained his necessary average shear coefficient of 0.37. For a horizontal section at the base of the dam, the shear coefficient would seem to be less than 0.37 for a safety factor of one, although that value must, of course, be appreciably exceeded for assured safety.

(4) It is difficult to obtain entirely dependable cohesion and friction factors by laboratory tests and both caution and good judgment are required in making, interpreting, and applying such tests to full-size embankment conditions. This is particularly so because laboratory tests are made on practically homogeneous soil samples, whereas embankments are rarely, if ever, entirely homogeneous. Laboratory tests, however, are indispensable aids in connection with earth-dam design and stability analyses, but their inherent limitations must be recognized and conservative safety factors must always be used in applying the results of such tests to actual design and construction.

(5) The author's several graphs of laboratory tests showing the relationship, under varying conditions, of shear coefficients, vertical pressures, penetration resistances, densities, etc., are all of great interest. Among other things they indicate the following:

(*a*) That density increases with vertical pressure but at a very much slower rate, and that under high vertical pressure the final densities are practically equal despite appreciable differences in the initial densities at which the samples were tested (Fig. 19(*b*));

(*b*) That, except for sample *D* for the higher pressures, there is a progressive decrease of the shear coefficient with increase of vertical pressure, the initial density remaining constant; and a progressive increase of the shear coefficient with increase of initial density, the vertical load remaining ing constant (Figs. 19(*c*) and 22); and

(*c*) That, except for sample *D* for the higher pressures, there is a progressive decrease of the shear coefficient with increase of penetration resistance, the initial density remaining constant; and a progressive increase of the shear coefficient with increase of initial density, the penetration resistance remaining constant (Fig. 20).

(6) Each of the indications cited, when considered alone, seems to be logical and, apparently, is in general accord with observations elsewhere; but a comparison of the shear coefficients for varying vertical pressures (Fig. 19(*c*)) with those for varying penetration resistances (Fig. 20), for identical arguments

in pounds per square inch, shows differences that are not so readily accounted for. These differences vary irregularly and range from zero to 70% or more, the higher shear values being those pertaining to penetration resistances. To be sure, vertical pressure and penetration resistance, as now determined, are not identical, and, although the latter is intended primarily as a quick field index of density and moisture content, yet it would seem, if it were accurate, that it also should be, in so far as shear values are concerned, a fair index of pressure equivalence even though its technique does differ from that of the normal pressure test.

(7) The preceding statement is not intended to imply that the laboratory tests were in any sense below par (apparently, they were all carefully made) but merely to indicate that such tests are not free of difficulties and limitations that may mar results, and also to emphasize that there is yet much that is elusive, vague, and unknown in the field of soils mechanics, particularly in the relationship of shear, internal friction, and cohesion. It is suspected that part of the cited difference in shear values is due to the fact that the present usual method of determining penetration resistance is not very accurate. Too much of the personal equation enters into that determination, involving, as it does, both the time rate and the intensity of man-power load application. Some new device should be developed that will largely eliminate the personal equation and permit more accurate correlation to density and supporting power of the soil. Such a device might take the form of a small fixed weight falling through a fixed height on a penetration needle of known diameter, and with a gage or scale calibration correlated to fairly large-size pressure tests on the same soil as that of the penetration needle tests. A series of such tests should afford a fairly dependable correlated calibration for manufacture of the apparatus.

(8) It is also possible that the generally accepted formula for shear may not be, and probably is not, entirely correct. It usually takes the form

$$F_t = C + f\,P \qquad (14)$$

in which F_t = total shear resistance; C = total cohesion resistance, or total resistance at zero loading, the cohesion per square foot being regarded as a constant for a given soil; f = coefficient of internal friction, regarded, usually, as a constant for a given soil; and P = total imposed vertical load.

It is the writer's concept that both total friction and total cohesion increase with density and with vertical loading; that the coefficients of friction and cohesion generally decrease with increase of vertical loading; and, as previously stated, that soil densities increase with vertical loading at a greatly retarded and progressively decelerating rate. On the basis of these concepts the shear formula might take the basic form,

$$F_t = C + c\,P + f\,P \qquad (15)$$

in which the additional factor, c, is the coefficient of cohesion. The thought is that C is a base constant for a given soil and for a specific standard dry density, say 100 lb per cu ft, and that c and f are not constants but are variable functions of P—probably exponential functions and not identical for c and f. Only extensive laboratory tests correlated to proper mathematical analysis can determine what these functions are and what the equation in its final form should be.

(9) *Roller Tests, Zone 3.*—Fig. 9 seems to indicate that the lower compaction costs resulted from the use of the heavier roller units with relatively few passes rather than from the lighter units with a larger number of passes. Apparently, the lowest unit cost, particularly if considered on the basis of cost per cubic yard, or cost per square foot per ton-pass of roller, was for eight passes of the tandem combination, consisting of one double unit, loaded with sand and water, and one double unit, unloaded. The indicated initial unit cost, however, is not necessarily the final single criterion as to the best roller combination to be adopted. The final dry densities desired, and those actually attained under the different tests, must also be considered, and no doubt were.

(10) It would be informative, and would add appreciably to the completeness of the record as presented in the paper, if the following data were added to the tabulation immediately preceding Fig. 9, and for each of the roller combinations listed: The average cost of compaction per cubic yard, the average dry density attained at optimum moisture and what that optimum was, and the average travel speed in miles per hour. It would also be of interest if the author, in his closure, would state just what roller combination, and just what number of passes were actually used, in the construction of Zone 3.

(11) *Densities.*—The densities attained in San Gabriel Dam are a good illustration of what can be accomplished with any reasonably good earth material under the present efficient methods of constant check of the properties of the materials as placed, and unremittent compaction and moisture control during the entire period of embankment construction. The degree of density obtained is largely a question of economics—what one feels warranted in expending for synthetic proportioning and mixing of the constituent materials, and, in compaction effort, to secure the density desired. The Zone 3 dry density of 144 lb per cu ft for material in place, including rock and fines, is perhaps 10% to 15% in excess of that which, in earlier years, was attained or attainable with the equipment then available, and then regarded as acceptably good. There are many intact old embankments whose dry densities are less than 120 lb per cu ft. With its moisture content of 5%, the wet density of the Zone 3 material in place was about 151 lb per cu ft, a density, as the author indicates, comparable to that of good concrete.

(12) Other recent examples of high densities attained in earth embankments are Green Mountain Dam on the Blue River in Colorado, and Deer Creek Dam on the Provo River in Utah, completed in 1941, both being structures of the U. S. Bureau of Reclamation. The former, for its impervious zone, attained an average dry density of about 141 lb per cu ft for the material in place and an average wet density at optimum moisture (±9%) of about 153 lb per cu ft, with departures from about 5% below to 2% above these values. For Deer Creek Dam, the average wet field density of the impervious section was also about 153 lb per cu ft with an optimum moisture somewhat greater

than that for Green Mountain Dam. These densities, in the main, were secured with twelve passes of duplex and triplex sheepsfoot roller units, ranging in weight from 1 ton per lin ft of roller, unloaded, to nearly 2 tons per lin ft fully sand and water loaded.

(13) *Seepage.*—The author states that the laboratory tests of Zone 3 material indicate a permeability coefficient of 1.05 (1.05 cu ft per sq ft per yr for unity gradient), a maximum velocity of 6.2 ft per yr (also for unity gradient), and a seepage loss of 99.5 cu ft per yr per lin ft of dam at its maximum section. This means, with reservoir water surface at spillway crest elevation, a total seepage for the structure of less than 0.01 cu ft per sec—an entirely negligible amount. If the data are available, a comparison of the seepage loss as indicated by laboratory test with that of the actual structure in operation at full reservoir stage would be of interest. The writer suspects that the latter is very much the greater.

(14) The fact is that for any acceptably good dam material, properly placed and compacted by modern methods, as already referred to in a previous paragraph, seepage, as indicated by laboratory test, is almost always negligibly small, and is almost always greatly exceeded by the actual seepage of the finished structure. This by no means implies that the laboratory work is poorly done, or that the dam is not well and carefully constructed, or that the actual seepages exceed safe permissibility although, in some cases, they may be of that character. Usually, the reverse of these conditions prevails.

(15) Seepage through the actual structure is not uniformly distributed; nor is it of uniform intensity, as through an entirely homogeneous mass of uniform density and porosity, such as is practically the case for laboratory tests. Usually, it manifests itself in distinctive areas or sections of the dam where, possibly, a minor shear plane may have developed from slight differential settlement, or where conditions as to compaction or composition of the material itself may have been somewhat below normal. Also, if adequate cutoffs have not been provided, the major seepage may be under or around the ends of the dam. Whatever the cause or causes, however, it may be anticipated that actual seepage will exceed laboratory seepage. It should be appreciated then, that, although laboratory tests are highly informative, indeed indispensable, and are valuable indexes of the comparative impermeabilities of the different borrow-pit materials from which selections must be made for dam construction, they should not be regarded as safe measures of the absolute seepage that will obtain in the finished structure.

Corrections for *Transactions:* In September, 1941, *Proceedings*, on page 1214, third from the last line, change "lines" to "fines"; in the abscissas of Fig. 13, change "sq in." to "cu ft"; and on page 1225, line beneath Eq. 5, change "$2\, g\, \mu$" to "$\sqrt{2\, g}\, \mu$."

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ENERGY LOSS AT THE BASE OF A FREE OVERFALL

Discussion

BY MERIT P. WHITE, ASSOC. M. AM. SOC. C. E.

MERIT P. WHITE,[9] Assoc. M. Am. Soc. C. E.[9a]—In this paper the author points out that flow conditions below an overfall are completely determined, provided the amount of energy lost in the transition is known. The author measures this energy loss in a model and gives a curve (Fig. 7) showing the relation between $\frac{E_1}{d_c}$ and $\frac{h}{d_c}$, in which E_1 is the total flow energy below the fall (expressed as head), d_c is the critical depth for parallel flow corresponding to the given discharge, and h is the drop in the floor of the channel (Fig. 5). It is shown that for large drops the energy loss is considerable.

Actually, this loss of energy can be determined fairly accurately by theory. This has been done, and the results are given in Fig. 15, which also shows, for comparison, the author's experimental curve (2) taken from Fig. 7.

The standing water behind the fall, shown in Fig. 5, results from the fact that a jet of fluid striking a surface tends to disperse in all directions along the surface. Fig. 16 shows a sheet of water striking a flat surface, which makes the angle θ with the jet. In this case there is nothing to interfere with the flow of water away from the point of impact. Here the greater part of the water flows to the right and the remainder to the left, as indicated by the dimensions d_a and d_b on the jet. Note that d_a and d_b are also the thicknesses of the water sheets flowing along the plane—that is, the water velocity changes only in direction, not in magnitude. The energy loss is negligible in this case. The ratio between d_a and d_b can be found from the fact that equal and opposite horizontal forces (parallel to the plane) act on the two water sheets to turn them from the original direction—that is, since the plane can only exert forces normal to itself, the net change in momentum parallel to the plane must be zero. Fig. 17 shows the velocity vectors for the upper water sheet before and after impact. In this figure the broken line shows ΔV, the change in velocity,

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NOTE.—This paper by Walter L. Moore, Jun. Am. Soc. C. E., was published in November, 1941, *Proceedings*.

[9] Asst. Prof., Illinois Inst. of Technology, Chicago, Ill.

[9a] Received by the Secretary November 24, 1941.

which is the difference between the two velocity vectors. The horizontal component of the velocity change equals $V(1 - \cos\theta)$. In the upper sheet the mass of fluid flowing per second in unit width is $\frac{\gamma}{g} V d_a$. Therefore, the change in the horizontal momentum in one second in unit width of channel must be $\frac{\gamma}{g} V^2 d_a (1 - \cos\theta)$ in the upper sheet, and $\frac{\gamma}{g} V^2 d_b (1 + \cos\theta)$ in the lower. Equating these gives

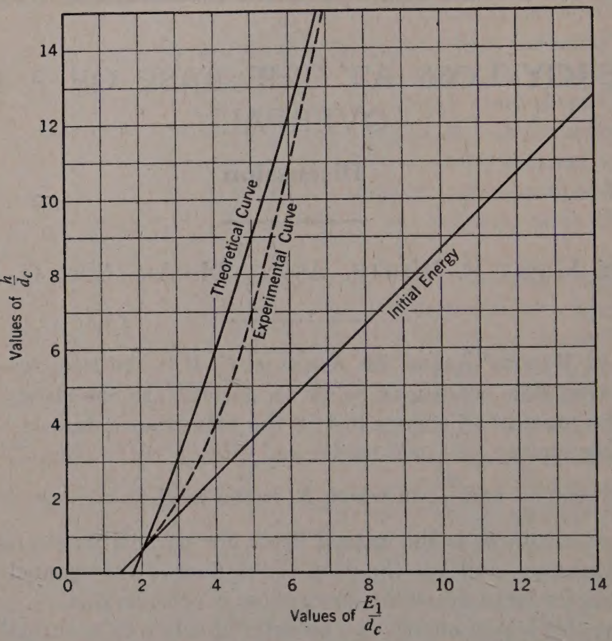

FIG. 15.—ENERGY AT THE BASE OF THE FALL (COMPARISON OF THEORETICAL VALUES WITH THE EXPERIMENTAL RESULTS SHOWN IN FIG. 7)

$$\frac{d_b}{d_a} = \frac{1 - \cos\theta}{1 + \cos\theta} \quad \text{(13)}$$

Because of the water standing beneath the fall (Fig. 5), the actual situation is somewhat different from that shown in Fig. 16, in which nothing hinders the flow of water along the plane. The water in the lower sheet flows into the bottom of the standing pool, causing clockwise rotation, while comparatively still water passes from the pool into the jet at exactly the same rate. This mixing is responsible for the loss of energy.

Fig. 18 outlines the general method of analysis and should be referred to frequently in the following: In the jet, just above the surface of the pool, there flows a quantity of water per second, Q, at a velocity V. Below the level of the surface of the pool the jet acquires additional fluid from the pool, which reduces the velocity of the jet. The jet must thicken here on account of the increase in Q and the decrease in V. At the floor the jet must divide into sheets of thicknesses d_a and d_b whose ratio depends on the angle of approach θ, according to Eq. 13. Since the flow to the right, away from the fall, must be Q, the flow to the left into the pool, Q_f, is determined in terms of θ—that is,

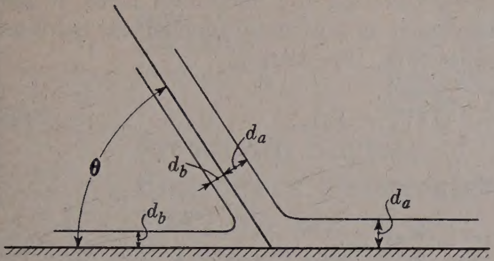

FIG. 16.—TWO-DIMENSIONAL JET STRIKING INCLINED PLANE

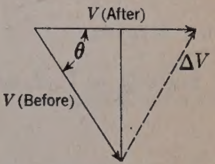

FIG. 17.—VELOCITIES IN UPPER SHEET OF JET BEFORE AND AFTER IMPACT

$$Q_f = \frac{d_b}{d_a} Q = \frac{1 - \cos\theta}{1 + \cos\theta} Q \qquad (14)$$

This is also the flow from the pool back into the jet. Assuming that this return flow has negligible momentum in the direction of the jet, the total momentum of the jet will not be changed in the mixing process. This determines V_m, the velocity after mixing. The momentum equation is

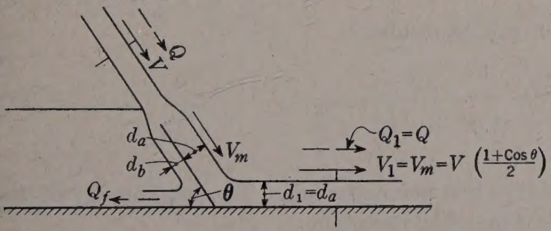

FIG. 18.—INCLINED JET WITH STANDING POOL ON ONE SIDE

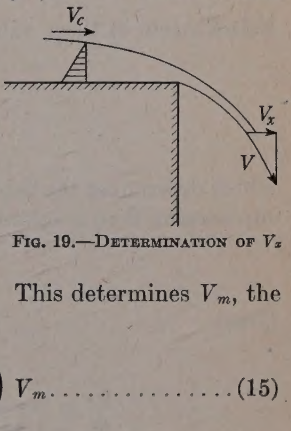

FIG. 19.—DETERMINATION OF V_x

$$\frac{\gamma}{g} Q V = \frac{\gamma}{g}\left(Q + Q\frac{1 - \cos\theta}{1 + \cos\theta}\right) V_m \qquad (15)$$

which gives

$$V_m = \frac{V}{2}(1 + \cos\theta) \qquad (16)$$

Since V_m equals V_1 (the outflow velocity):

$$V_1 = \frac{V}{2}(1 + \cos\theta) \qquad (17)$$

and the corresponding depth of flow is

$$d_1 = \frac{Q}{V_1} \qquad (18)$$

In these equations, V is the velocity and θ is the angle of inclination that the jet would have at the floor if there were no pool. The cosine of the angle θ is the ratio of the horizontal component of the jet velocity, V_x, to the total velocity V. Velocity V_x is found by equating the horizontal force at the critical section above the fall to the change in horizontal momentum between this point and any point in the free jet (Fig. 19)—that is,

$$\frac{\gamma}{2} d^2_c = \frac{\gamma}{g} Q (V_x - V_c) \qquad (19)$$

Since $Q = V_c d_c$ and $V_c = \sqrt{g\, d_c}$, this gives

$$V_x = \frac{3}{2} V_c \qquad (20)$$

Velocity V, the velocity corresponding to the total fall, is obtained from the initial energy.

$$V = \sqrt{2\, g \left(h + \frac{3}{2} d_c \right)} \qquad (21)$$

Then

$$\cos \theta = \frac{1.5\, V_c}{\sqrt{2\, g \left(h + \frac{3}{2} d_c \right)}} = \frac{1.06}{\sqrt{\frac{h}{d_c} + \frac{3}{2}}} \qquad (22)$$

Substitution of these values in Eq. 18 results in

$$\frac{d_1}{d_c} = \frac{\sqrt{2}}{1.06 + \sqrt{\frac{h}{d_c} + \frac{3}{2}}} \qquad (23)$$

which determines the depth of flow just below the fall. The total energy of the discharging fluid is found by means of the relation

$$E_1 = d_1 + \frac{V^2_1}{2\, g} \qquad (24)$$

Then

$$\frac{E_1}{d_c} = \frac{\sqrt{2}}{1.06 + \sqrt{\frac{h}{d_c} + \frac{3}{2}}} + \frac{\left(1.06 + \sqrt{\frac{h}{d_c} + \frac{3}{2}} \right)^2}{4} \qquad (25)$$

The solid line of Fig. 15 shows this relation, and the broken line gives the results of the author's experimental investigation. In view of the crudeness of the present analysis the agreement between theory and measurement is satisfactory.